Math 3300 Review sheet
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Problem 1. Let

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

(1) Orthogonally diagonalize A.
(2) Write the quadratic form given by A. Is it positive definite or negative definite or indefinite?

Problem 2. Find a basis for the orthogonal complement of

$$W = \text{Span}\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \}$$

Problem 3. Let W be a subspace of $\mathbb{R}^4$ given by a basis

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ -1 \\ 2 \end{pmatrix}$$

Find an orthonormal basis for W.

Problem 4. Let

$$W = \text{Span}\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \}, \quad \mathbf{v} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

Find the orthogonal projection of $\mathbf{v}$ onto W.

Problem 5. Let

$$f_1(t) = 1 + t^3, \quad f_2(t) = t^2 - 1. \quad f_3(t) = 2t^3 - t + 1$$

be vectors in $\mathbb{P}_3$. Determine whether they are linearly independent.

Problem 6. Prove that

$$W = \{f(t) \in \mathbb{P}_3 : f(2) - 2f(3) = 0\}$$

is a subspace of $\mathbb{P}_3$.

More problems on the back!

Problem 7. Given that

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$$

are a basis for $\mathbb{R}^3$. Find the coordinate vector of $\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$ relative to this basis.

Problem 8. Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{pmatrix}$$

Problem 9. Let T be a map from $\mathbb{P}_2$ to $\mathbb{R}^2$ given by

$$T(f) = \begin{pmatrix} f(1) + f(2) \\ 2f(3) \end{pmatrix}$$

Prove that T is a linear transformation.