

Total time: 10 minutes.

Problem 1 (10 points). Determine whether the following matrix is diagonalizable. If yes, diagonalize it; if no, explain why.

$$A = \begin{pmatrix} -1 & 4 \\ -1 & 3 \end{pmatrix}$$

$$\det(A - \lambda I) = \det \begin{pmatrix} -1 - \lambda & 4 \\ -1 & 3 - \lambda \end{pmatrix} = (-1 - \lambda)(3 - \lambda) + 4 = \lambda^2 - 2\lambda + 1 = (\lambda - 1)^2$$

Therefore $\det(A - \lambda I) = 0$ gives one root $\lambda = 1$.

To find its corresponding eigenspace, solve $(A - \lambda I)\mathbf{x} = 0$.

$$\begin{pmatrix} -2 & 4 \\ -1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 4 \\ 0 & 0 \end{pmatrix}$$

There is one free variable x_2 , and thus the eigenspace has dimension 1. Since A is 2×2 and $1 < 2$, A is not diagonalizable.

(here one eigenvector is $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ but we don't need to calculate it.)