Total time: 10 minutes.

Problem 1 (10 points). Find the eigenvalues and the corresponding eigenspaces for the following matrix:

$$\begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}$$

Call this matrix A.

$$\det(A-\lambda I) = \det\begin{pmatrix} -2-\lambda & 3 \\ 1 & -\lambda \end{pmatrix} = (-2-\lambda)(-\lambda) - 3 = \lambda^2 + 2\lambda - 3 = (\lambda+3)(\lambda-1)$$

$\det(A-\lambda I) = 0$ gives the eigenvalues $\lambda_1 = -3$, $\lambda_2 = 1$.
For $\lambda_1 = -3$, solve $(A - \lambda_1 I)\mathbf{x} = 0$:

$$\begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} \underline{1} & 3 \\ 0 & 0 \end{pmatrix}$$

Solutions are

$$\begin{cases} x_1 = -3x_2 \\ x_2 \text{ free} \end{cases}, \quad \mathbf{x} = x_2 \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

Therefore the corresponding eigenspace is $\text{Span}\{\begin{pmatrix} -3 \\ 1 \end{pmatrix}\}$.
For $\lambda_2 = 1$, solve $(A - \lambda_2 I)\mathbf{x} = 0$:

$$\begin{pmatrix} -3 & 3 \\ 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 3 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \underline{1} & -1 \\ 0 & 0 \end{pmatrix}$$

Solutions are

$$\begin{cases} x_1 = x_2 \\ x_2 \text{ free} \end{cases}, \quad \mathbf{x} = x_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Therefore the corresponding eigenspace is $\text{Span}\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\}$.