

Total time: 10 minutes.

Problem 1 (10 points). Calculate the determinant of the following matrix.

$$\begin{pmatrix} -1 & -1 & -6 & 3 \\ 2 & 0 & 4 & -1 \\ 0 & 0 & 0 & 3 \\ 1 & 0 & 4 & -4 \end{pmatrix}$$

$$\det \begin{pmatrix} -1 & -1 & -6 & 3 \\ 2 & 0 & 4 & -1 \\ 0 & 0 & 0 & 3 \\ 1 & 0 & 4 & -4 \end{pmatrix}$$

$$= (-1)^{3+4} \cdot 3 \cdot \det \begin{pmatrix} -1 & -1 & -6 \\ 2 & 0 & 4 \\ 1 & 0 & 4 \end{pmatrix} \quad (\text{expand across row 3})$$

$$= -3 \cdot (-1)^{1+2} \cdot (-1) \det \begin{pmatrix} 2 & 4 \\ 1 & 4 \end{pmatrix} \quad (\text{expand across column 2})$$

$$= -3 \cdot (8 - 4) = -12$$

(you can also do other ways of row/column expansions or use row reductions)