

Total time: 10 minutes.

Problem 1 (10 points). Let

$$A = \begin{pmatrix} -2 & -1 & -6 & 3 \\ 2 & 1 & 4 & -1 \end{pmatrix}$$

- (1) Find a basis for Col A .
- (2) Find a basis for Nul A .

Row operations to make A into reduced echelon:

$$\begin{pmatrix} -2 & -1 & -6 & 3 \\ 0 & 0 & -2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1/2 & 3 & -3/2 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} \underline{1} & 1/2 & 0 & 3/2 \\ 0 & 0 & \underline{1} & -1 \end{pmatrix}$$

Pivots are underlined. They are in the first and third columns. Therefore these columns in A form a basis for Col A , that is, $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -6 \\ 4 \end{pmatrix}$.

The general solution to $A\mathbf{x} = 0$ can be written as

$$\begin{cases} x_1 = -\frac{1}{2}x_2 - \frac{3}{2}x_4 \\ x_2 \text{ free} \\ x_3 = x_4 \\ x_4 \text{ free} \end{cases}$$

or in parametric form,

$$\mathbf{x} = x_2 \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -\frac{3}{2} \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

Therefore a basis for Nul A is $\begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{3}{2} \\ 0 \\ 1 \\ 1 \end{pmatrix}$.