

Total time: 10 minutes.

Problem 1 (10 points). Solve the equation $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 0 & 1 & 3 & -2 \\ -1 & 1 & 2 & 5 \\ 2 & -3 & -7 & -8 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$$

The augmented matrix is

$$\begin{pmatrix} 0 & 1 & 3 & -2 & 2 \\ -1 & 1 & 2 & 5 & 1 \\ 2 & -3 & -7 & -8 & -4 \end{pmatrix}$$

switch row1 and row2

$$\begin{pmatrix} -1 & 1 & 2 & 5 & 1 \\ 0 & 1 & 3 & -2 & 2 \\ 2 & -3 & -7 & -8 & -4 \end{pmatrix}$$

2*row1 added to row3

$$\begin{pmatrix} -1 & 1 & 2 & 5 & 1 \\ 0 & 1 & 3 & -2 & 2 \\ 0 & -1 & -3 & 2 & -2 \end{pmatrix}$$

1*row2 added to row3

$$\begin{pmatrix} -1 & 1 & 2 & 5 & 1 \\ 0 & 1 & 3 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

-1*row2 added to row1

$$\begin{pmatrix} -1 & 0 & -1 & 7 & -1 \\ 0 & 1 & 3 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

row1 multiplied by -1

$$\begin{pmatrix} 1 & 0 & 1 & -7 & 1 \\ 0 & 1 & 3 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Therefore the solution is

$$\begin{cases} x_1 = 1 - x_3 + 7x_4 \\ x_2 = 2 - 3x_3 + 2x_4 \\ x_3 \text{ free} \\ x_4 \text{ free} \end{cases}$$