Math 2250 Quiz 8 (20221101) Solution

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Total time: 15 minutes.

Problem 1 (2 points). Determine limit.

$$\lim_{x\to\pi}\frac{\sin(2x)}{\pi^{1/3}-x^{1/3}}=\lim_{x\to\pi}\frac{2\cos(2x)}{-\frac{1}{3}x^{-2/3}}=\frac{2\cos(2\pi)}{-\frac{1}{3}\pi^{-2/3}}=-6\pi^{2/3}$$

(0/0 type l'Hopital)

Problem 2 (8 points). A rectangle has area 4. What is the smallest possible length of its diagonal?

Denote x and y as the two sides of the rectangle. Then we know $xy=4$ and want to minimize

$$f=\sqrt{x^2+y^2}$$

(by Pythagoras). We have $y=4/x$. Plug into above,

$$f(x)=\sqrt{x^2+(\frac{4}{x})^2}=\sqrt{x^2+\frac{16}{x^2}}$$

The domain of $f(x)$ is $(0,\infty)$.

Taking derivative,

$$f'(x)=\frac{1}{2}(x^2+\frac{16}{x^2})^{-1/2}\cdot(2x-\frac{32}{x^3})$$

Setting $f'(x)=0$ gives

$$2x-\frac{32}{x^3}=0,\quad x^4=16,\quad x=\pm 2$$

The root $x=-2$ is discarded because it is not in the domain. Therefore we get one critical point $x=2$.

$$f(2)=\sqrt{8},\quad \lim_{x\to 0^+}f(x)=\lim_{x\to\infty}f(x)=\infty$$

Therefore the global minimum of f on $(0,\infty)$ is achieved at $f(2)=\sqrt{8}$. Therefore the smallest possible length of the diagonal is $\sqrt{8}$.