

Total time: 15 minutes.

Problem 1 (3 points each). Calculate definite integrals:

$$(1) \int_1^2 \frac{x^2 - 1}{x} dx = \int_1^2 \left(x - \frac{1}{x}\right) dx = \left(\frac{1}{2}x^2 - \ln|x|\right)\Big|_1^2 = (2 - \ln 2) - \left(\frac{1}{2} - \ln 1\right) = \frac{3}{2} - \ln 2$$

$$(2) \int_{-4}^0 \sqrt{4 - (x + 2)^2} dx$$

The function is $y = \sqrt{4 - (x + 2)^2}$, $(x + 2)^2 + y^2 = 4$. This is a circle centered at $(-2, 0)$ with radius 2, and the integral calculates the area of its upper half. Therefore the result is $\frac{1}{2}\pi \cdot 2^2 = 2\pi$.

Problem 2 (4 points). Water is flowing into a tank at a rate of $(t^2 + 1)$ gallons per minute (where t is the time variable). If initially there is 2 gallons of water in the tank, what is the amount of water in the tank after 3 minutes?

$$2 + \int_0^3 (t^2 + 1) dt = 2 + \left(\frac{1}{3}t^3 + t\right)\Big|_0^3 = 2 + (9 + 3) - 0 = 14$$