1. (1) $\int x^2 e^{x^3}\,dx = \frac{1}{3}\int 3x^2 e^{x^3}\,dx$

$u = x^3$

$du = 3x^2\,dx$

$= \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3} + C$

(2) $\int \sin(\sin x)\cos x\,dx = \int \sin u\,du = -\cos u + C$

$u = \sin x$

$= -\cos(\sin x) + C$

$du = \cos x\,dx$

(3) $\int \frac{x}{\sqrt{3-12x^2}}\,dx = -\frac{1}{24}\int \frac{-24x}{\sqrt{3-12x^2}}\,dx$

$u = 3-12x^2$

$= -\frac{1}{24}\int \frac{1}{\sqrt{u}}\,du$

$du = -24x\,dx$

$= -\frac{1}{24}2\sqrt{u} + C$

$= -\frac{1}{12}\sqrt{3-12x^2} + C$

(4) $\int \frac{3}{\sqrt{3-12x^2}}\,dx = 3\int \frac{1}{\sqrt{3(1-4x^2)}}\,dx$

$= \frac{3}{\sqrt{3}}\int \frac{1}{\sqrt{1-4x^2}}\,dx$

$u = 2x$

$du = 2\,dx$

$$= \frac{\sqrt{3}}{2} \int \frac{2}{\sqrt{1-4x^2}}\, dx$$

$$= \frac{\sqrt{3}}{2} \int \frac{1}{\sqrt{1-u^2}}\, du$$

$$= \frac{\sqrt{3}}{2} \sin^{-1} u + C = \frac{\sqrt{3}}{2} \sin^{-1}(2x) + C$$

(5) $\int \sin x \cos^4 x \, dx = -\int (-\sin x) \cos^4 x \, dx$

$u = \cos x$ $\qquad = -\int u^4 \, du$

$du = -\sin x \, dx$ $\qquad = -\frac{1}{5} u^5 + C$

$$= -\frac{1}{5} \cos^5 x + C$$

(6) $\int x \sin^2(x^2)\, dx = \frac{1}{2} \int 2x \sin^2(x^2)\, dx$

$u = x^2$ $\qquad = \frac{1}{2} \int \sin^2 u \, du$

$du = 2x\,dx$

$$= \frac{1}{2} \int \frac{1}{2} (1 - \cos 2u)\, du$$

$$= \frac{1}{4} \int (1 - \cos 2u)\, du$$

$$= \frac{1}{4} \left(u - \frac{1}{2} \sin 2u \right) + C$$

$$= \frac{1}{4} \left(x^2 - \frac{1}{2} \sin(2x^2) \right) + C$$

1.) $\displaystyle\int \frac{1}{x\,(1+2(\ln x)^2)}\,dx = \int \frac{1}{1+2u^2}\,du$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \frac{1}{\sqrt{2}} \int \frac{\sqrt{2}}{1+2u^2}\,du$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{1+v^2}\,dv$$

$v^2 = 2u^2$

$v = \sqrt{2}\,u$

$dv = \sqrt{2}\,du$

$$= \frac{1}{\sqrt{2}} \tan^{-1} v + C$$

$$= \frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2}\,u) + C$$

$$= \frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2}\ln x) + C$$

2. $\displaystyle\frac{d}{dx}\int_{\sin x}^{x^2} \frac{1}{1+t^3}\,dt = \frac{1}{1+x^6}\cdot 2x - \frac{1}{1+\sin^3 x}\cdot\cos x$

$a(x) = \sin x,\ b(x) = x^2,\ f(t) = \frac{1}{1+t^3}$

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\,dt = f(b(x))\cdot b'(x) - f(a(x))\cdot a'(x)$$

3. (1) $\displaystyle\int_0^1 e^x \cos(e^x)\,dx = \int_1^e \cos u\,du = \sin u\Big|_1^e$

$u = e^x$ $\quad x: [0,1]$

$du = e^x\,dx$ $\quad u: [1,e]$

$= \sin(e) - \sin(1)$

(2) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{3+\sin x} dx = \int_3^4 \frac{1}{u} du = \ln|u| \Big|_3^4$

$u = 3+\sin x$ $\quad$ $x : [0, \frac{\pi}{2}]$ $\quad$ $= \ln 4 - \ln 3$

$du = \cos x \, dx$ $\quad$ $u : [3, 4]$

(3) $\int_0^1 (3x^2+1)\sqrt{x^3+x} \, dx = \int_0^2 \sqrt{u} \, du = \frac{2}{3} u^{\frac{3}{2}} \Big|_0^2$

$u = x^3 + x$ $\quad$ $x: [0, 1]$ $\quad$ $= \frac{2}{3} \cdot 2^{\frac{3}{2}}$

$du = (3x^2+1)dx$ $\quad$ $u: [0, 2]$