1. $\int_1^2 x^2\,dx$ by definition

$$x_0 = 1, \quad \Delta x = \frac{2-1}{n} = \frac{1}{n}, \quad x_i = 1 + \frac{i}{n}$$

$$\sum_{i=1}^{n} f(x_i)\cdot\Delta x = \sum_{i=1}^{n} \left(1+\frac{i}{n}\right)^2 \cdot \frac{1}{n}$$

$$= \sum_{i=1}^{n} \left(1 + \frac{2i}{n} + \frac{i^2}{n^2}\right)\frac{1}{n}$$

$$= \frac{1}{n}\underbrace{\sum_{i=1}^{n} 1}_{\underbrace{1+1+\cdots+1}_{n\text{ times}}} + \frac{2}{n^2}\sum_{i=1}^{n} i + \frac{1}{n^3}\sum_{i=1}^{n} i^2$$

$$= \frac{1}{n}\cdot n + \frac{2}{n^2}\cdot\frac{1}{2}n(n+1) + \frac{1}{n^3}\cdot\frac{1}{6}n(n+1)(2n+1)$$

$$= 1 + \frac{n+1}{n} + \frac{2n^2+3n+1}{6n^2}$$

$$\xrightarrow{(n\to\infty)} 1 + 1 + \frac{1}{3} = \frac{7}{3}$$

2. (1) $\int_0^{\pi} (\sin x + \cos x)\, dx$

$= (-\cos x + \sin x)\Big|_0^{\pi}$

$= (-\cos\pi + \sin\pi) - (-\cos 0 + \sin 0)$

$= (-(-1) + 0) - (-1 + 0) = 2$

(2) $\int_{-1}^{1} (x^2 - 2e^x)\, dx$

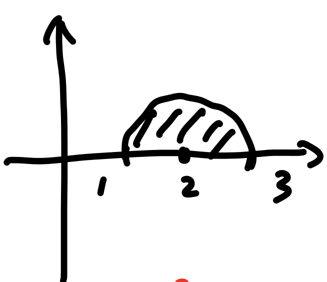
$= \left(\frac{1}{3}x^3 - 2e^x\right)\Big|_{-1}^{1}$

$= \left(\frac{1}{3}\cdot 1^3 - 2e^1\right) - \left(\frac{1}{3}\cdot(-1)^3 - 2e^{-1}\right)$

$= \frac{2}{3} - 2e + 2e^{-1}$

(3) $\int_1^3 \sqrt{1-(x-2)^2}\, dx$

$= \frac{1}{2}\cdot\pi\cdot 1^2 = \boxed{\frac{\pi}{2}}$



$y = \sqrt{1-(x-2)^2}$

$y^2 = 1-(x-2)^2$

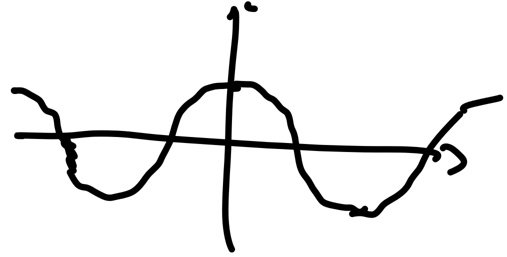
$(x-2)^2 + y^2 = 1$

center $(2,0)$, radius 1

$(x-a)^2 + (y-b)^2 = R^2$

is a circle centered at (a,b) w/ radius R.

(4) $f(x) = x(\cos x)^3$

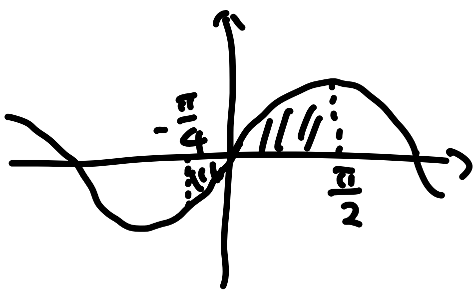


$$f(-x) = (-x)\left(\cos(-x)\right)^3$$

$$= -x(\cos x)^3 = -f(x)$$

$\Rightarrow$ f is odd. $\Rightarrow$ $\int_{-1}^{1} x\cos^3 x\,dx = 0$

3. $y = \sin x$ on $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right]$



$$\text{Total area} = -\int_{-\frac{\pi}{4}}^{0} \sin x\,dx + \int_{0}^{\frac{\pi}{2}} \sin x\,dx$$

$$= +(+\cos x)\Big|_{-\frac{\pi}{4}}^{0} + (-\cos x)\Big|_{0}^{\frac{\pi}{2}}$$

$$= \cos 0 - \cos\left(-\frac{\pi}{4}\right) + \left(-\cos\frac{\pi}{2}\right) - (-\cos 0)$$

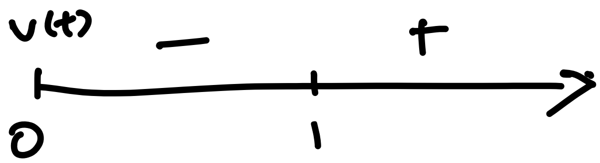
$$= 1 - \frac{\sqrt{2}}{2} + 1 = 2 - \frac{\sqrt{2}}{2}.$$

4. $v(t) = t^2 - t = t(t-1)$

(1) $\int_0^2 (t^2 - t)\,dt = \left(\frac{1}{3}t^3 - \frac{1}{2}t^2\right)\Big|_0^2$

$$= \left(\frac{1}{3}\cdot 2^3 - \frac{1}{2}\cdot 2^2\right) - 0 = \frac{2}{3}$$

(2)



Total distance traveled $= -\int_0^1 (t^2 - t)\,dt + \int_1^2 (t^2 - t)\,dt$

$= 1$

Total gas $= 0.1 \times 1 = 0.1$.