1. (1) $\lim_{x\to0^+}(\sin x)^x = \lim_{x\to0^+} e^{\ln(\sin x)^x}$

$= \lim_{x\to0^+} e^{x\ln(\sin x)} = e^0 = \boxed{1}$

$$\lim_{x\to0^+} x\ln(\sin x) = \lim_{x\to0^+}\frac{\ln(\sin x)}{\frac{1}{x}} \underset{\uparrow\ \text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to0^+}\frac{\frac{1}{\sin x}\cdot\cos x}{-\frac{1}{x^2}}$$

$$= \lim_{x\to0^+}\frac{x^2\cos x}{-\sin x} \underset{\uparrow\ \text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to0^+}\frac{2x\cos x + x^2\cdot(-\sin x)}{-\cos x} = 0$$

(2) $\lim_{x\to\infty} x^{\frac{1}{\ln(\ln x)}} = \lim_{x\to\infty} e^{\ln x^{\frac{1}{\ln(\ln x)}}} = \lim_{x\to\infty} e^{\frac{1}{\ln(\ln x)}\cdot\ln x}$

$= \boxed{\infty}$

$$\lim_{x\to\infty}\frac{\ln x}{\ln(\ln x)} \underset{\uparrow\ \text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty}\frac{\cancel{\frac{1}{x}}}{\frac{1}{\ln x}\cdot\cancel{\frac{1}{x}}} = \lim_{x\to\infty}\ln x = \infty$$

(3) $\lim_{x\to1} x^{\frac{1}{1-x}} = \lim_{x\to1} e^{\ln x^{\frac{1}{1-x}}} = \lim_{x\to1} e^{\frac{1}{1-x}\ln x}$

$= \boxed{e^{-1}}$

$$\lim_{x\to1}\frac{\ln x}{1-x} \underset{\uparrow\ \text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to1}\frac{\frac{1}{x}}{-1} = -1$$

2. (1) $\int (x^3 - x)^2 \, dx$

$= \int (x^6 - 2x^4 + x^2) \, dx$

$= \frac{1}{7}x^7 - 2 \cdot \frac{1}{5}x^5 + \frac{1}{3}x^3 + C$

(2) $\int \left(2\cos x + 3e^x - \frac{2}{1+x^2}\right) dx$

$= 2\sin x + 3e^x - 2 \cdot \tan^{-1} x + C$

(3) $\int \frac{x^3 + x^2 \sin x - x + 4}{x^2} \, dx$

$= \int (x + \sin x - x^{-1} + 4x^{-2}) \, dx$

$= \frac{1}{2}x^2 - \cos x - \ln|x| - 4x^{-1} + C$

(4) $\int e^{2x} \, dx = \frac{1}{2}e^{2x} + C$

$(e^{2x})' = e^{2x} \cdot 2$

(5) $\int (1 + e^x)(2 - e^{2x}) \, dx$

$= \int (2 - e^{2x} + 2e^x - e^{3x}) \, dx$

$= 2x - \frac{1}{2}e^{2x} + 2e^x - \frac{1}{3}e^{3x} + C$

3. $f(x) = 2x - x^2$ on $[0, 2]$

(1) right endpoint approx., $n = 4$

$$x_0 = 0, \quad \Delta x = \frac{2-0}{4} = \frac{1}{2} \qquad x_i = 0 + i \cdot \Delta x = \frac{i}{2}$$

$$\text{Area} \approx \sum_{i=1}^{4} f(x_i)\Delta x$$

$$= f\left(\tfrac{1}{2}\right)\cdot\Delta x + f(1)\cdot\Delta x + f\left(\tfrac{3}{2}\right)\cdot\Delta x + f(2)\Delta x$$

$$= \frac{3}{4}\cdot\frac{1}{2} + 1\cdot\frac{1}{2} + \frac{3}{4}\cdot\frac{1}{2} + 0\cdot\frac{1}{2} = \frac{5}{4}$$

(2) right endpoint approx., $n = 100$

$$x_0 = 0, \quad \Delta x = \frac{2-0}{100} = \frac{2}{100}, \quad x_i = 0 + i\frac{2}{100}$$
$$= \frac{2}{100} i$$

$$\text{Area} \approx \sum_{i=1}^{100} f(x_i)\Delta x = \sum_{i=1}^{100}\left(2\cdot\frac{2}{100}i - \left(\frac{2}{100}i\right)^2\right)\cdot\frac{2}{100}$$

$$= \frac{8}{10000}\sum_{i=1}^{100} i - \left(\frac{2}{100}\right)^3 \sum_{i=1}^{100} i^2$$

$$= \frac{8}{10000}\cdot\frac{1}{2}\cdot 100\cdot 101 - \left(\frac{2}{100}\right)^3\cdot\frac{1}{6}\cdot 100\cdot 101\cdot 201 = 1.3332$$

(3) $\frac{4}{3}$

(General right endpoint approx:

$$\sum_{i=1}^{n}\left(2\cdot\frac{2}{n}i - \left(\frac{2}{n}i\right)^2\right)\cdot\frac{2}{n} = \frac{8}{n^2}\sum_{i=1}^{n} i - \frac{8}{n^3}\sum_{i=1}^{n} i^2$$

$$= \frac{8}{n^2} \cdot \frac{1}{2} n(n+1) - \frac{8}{n^3} \cdot \frac{1}{6} n(n+1)(2n+1)$$

$$\xrightarrow{\text{as } n \to \infty} \frac{8}{2} - \frac{8}{6} \cdot 2 = \frac{4}{3}$$