MATH 2250 QUIZ 6 (20221011) SOLUTION
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Total time: 15 minutes.

Problem 1 (4+2 = 6 points). Let $f(x) = \tan^{-1} x$.
(1) Find the linear approximation of f at $x = 1$.
(2) Use your result in (1) to approximate $\tan^{-1} 0.999$.

(1)

$$f'(x) = \frac{1}{1+x^2}, \quad f(1) = \frac{\pi}{4}, \quad f'(1) = \frac{1}{2}$$

Therefore the linear approximation is

$$L(x) = \frac{\pi}{4} + \frac{1}{2}(x-1)$$

(2)

$$\tan^{-1} 0.999 = f(0.999) \approx L(0.999) = \frac{\pi}{4} + \frac{1}{2}(0.999 - 1) = \frac{\pi}{4} - 0.0005$$

Problem 2 (4 points). Find the global maximum and minimum of $f(x) = \frac{x}{4+x^2}$ on $[0, 3]$.

$$f'(x) = \frac{1 \cdot (4+x^2) - x \cdot 2x}{(4+x^2)^2} = \frac{4-x^2}{(4+x^2)^2}$$

$$f'(x) = 0 \quad \Rightarrow \quad 4 - x^2 = 0 \quad \Rightarrow \quad x = 2, -2$$

The root $x = -2$ is discarded because it is not in the interval $[0, 3]$. Therefore we get the critical point $x = 2$.

$$f(2) = \frac{1}{4} = 0.25, \quad f(0) = 0, \quad f(3) = \frac{3}{13} = 0.23\cdots$$

Therefore the global maximum is $f(2) = \frac{1}{4}$, the global minimum is $f(0) = 0$.