$$\lim_{x\to\infty} \frac{e^x - 1}{e^{2x}+2} = \lim_{x\to\infty} \frac{1 - \frac{1}{e^x}}{e^x + \frac{2}{e^x}} = 0$$

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1. (1) $$\lim_{x\to 2} \frac{\overbrace{x^2 - 4}^{\to 0}}{\underbrace{x^{1/3} - 8}_{\to 2^{1/3}-8}} = 0$$

(2) $$\lim_{x\to\pi} \frac{\cos x + 1}{\sqrt{x} - \sqrt{\pi}} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\uparrow}{=}} \lim_{x\to\pi} \frac{-\sin x}{\frac{1}{2}x^{-\frac{1}{2}}} = 0$$

(3) $$\lim_{x\to\frac{\pi}{2}} \frac{\sin x - 1}{x - \pi} = 0$$

(4) $$\lim_{x\to 0} \frac{\cos x - 1}{x^2} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\uparrow}{=}} \lim_{x\to 0} \frac{-\sin x}{2x} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\uparrow}{=}} \lim_{x\to 0} \frac{-\cos x}{2}$$

$$= -\frac{1}{2}$$

(5) $$\lim_{x\to\infty} \frac{x + \sin x}{x - \cos x} \underset{\text{"}\frac{\infty}{\infty}\text{"}}{\overset{\uparrow}{=}} \lim_{x\to\infty} \frac{1 + \cos x}{1 + \sin x}$$ DNE

↳ l'Hopital doesn't apply

$$\lim_{x\to\infty} \frac{x + \sin x}{x - \cos x} = \lim_{x\to\infty} \frac{1 + \overbrace{\frac{\sin x}{x}}^{\to 0}}{1 - \underbrace{\frac{\cos x}{x}}_{\to 0}} = 1$$

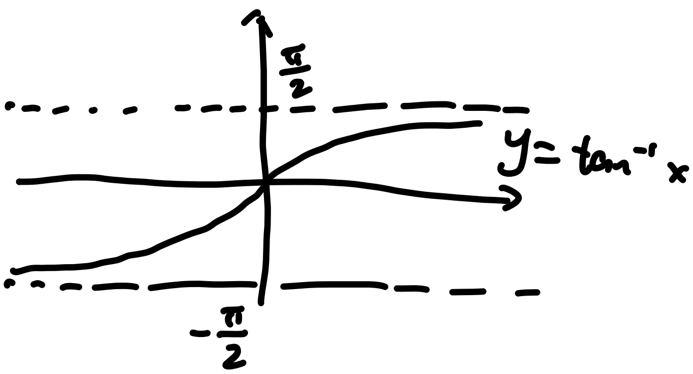
(6) $$\lim_{x\to\infty} \frac{\ln x}{\ln(\ln x)} \underset{\uparrow \; \text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{\cancel{\frac{1}{x}}}{\frac{1}{\ln x}\cdot \cancel{\frac{1}{x}}} = \lim_{x\to\infty} \frac{1}{\frac{1}{\ln x}}$$

$$= \lim_{x\to\infty} \ln x = \infty .$$

(7) $$\lim_{x\to 0^+} \underbrace{x}_{\downarrow\, 0}\, \underbrace{\ln x}_{\downarrow\, -\infty} = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \underset{\uparrow \; \text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0^+} \frac{1}{-\frac{1}{x}} = 0$$

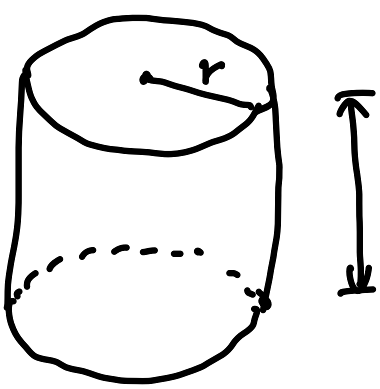
(8) $$\lim_{x\to\infty} \underbrace{x}_{\searrow\infty}\underbrace{\left(\tan^{-1}x - \frac{\pi}{2}\right)}_{\searrow 0} = \lim_{x\to\infty} \frac{\tan^{-1}x - \frac{\pi}{2}}{\frac{1}{x}}$$



$$\underset{\uparrow \; \text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to\infty} \frac{\frac{1}{1+x^2}\cdot(-x^2)}{-\frac{1}{x^2}\quad(-x^2)}$$

$$= \lim_{x\to\infty} \frac{-x^2}{1+x^2} = -1$$

2.



know $\pi r^2 h = 4$

want minimize

$$A = 2\pi r h + 2\pi r^2$$

$$h = \frac{4}{\pi r^2}$$

$$A(r) = 2\pi r \cdot \frac{4}{\pi r^2} + 2\pi r^2 = \frac{8}{r} + 2\pi r^2$$

domain : $r \in (0, \infty)$

$$A'(r) = -\frac{8}{r^2} + 4\pi r = 0$$

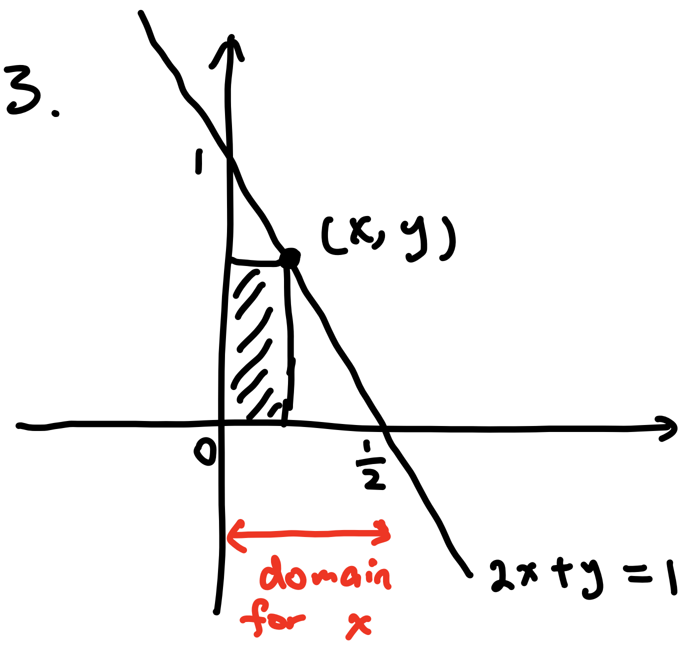
$$\frac{8}{r^2} = 4\pi r \qquad r^3 = \frac{8}{4\pi} = \frac{2}{\pi}$$

$$r = \left(\frac{2}{\pi}\right)^{1/3}$$

$$A\left(\left(\frac{2}{\pi}\right)^{1/3}\right) = \boxed{\frac{8}{\left(\frac{2}{\pi}\right)^{1/3}} + 2\pi\left(\frac{2}{\pi}\right)^{2/3}} \longrightarrow \text{global min.}$$

$$\lim_{r \to 0^+} A(r) = \infty, \qquad \lim_{r \to \infty} A(r) = \infty$$

3.



know: $2x + y = 1$

want max : $A = xy$

$y = 1 - 2x$

$$A(x) = x(1-2x) = x - 2x^2$$

domain : $x \in [0, \frac{1}{2}]$

$$A'(x) = 1 - 4x = 0 \Rightarrow x = \frac{1}{4}$$

$$A\left(\frac{1}{4}\right) = \boxed{\frac{1}{8}}, \quad A(0) = A\left(\frac{1}{2}\right) = 0$$

global max.