1. (1) $$\lim_{x\to\infty} \frac{3x^2+x^3-2}{2x^2-5} = \lim_{x\to\infty} \frac{3+x-\frac{2}{x^2}}{2-\frac{5}{x^2}} = \infty$$

(2) $$\lim_{x\to-\infty} \frac{x^2+x-3}{2x^2+3\sin(2x)}$$

$$= \lim_{x\to-\infty} \frac{1+\frac{1}{x}-\frac{3}{x^2}}{2+3\cdot\frac{\sin(2x)}{x^2}} = \frac{1}{2}$$

(3) $$\lim_{x\to\infty} \frac{2x+3}{\sqrt{3x^2-x-2}}$$

$$= \lim_{x\to\infty} \frac{2+\frac{3}{x}}{\sqrt{\frac{1}{x^2}}\sqrt{3x^2-x-2}}$$

$$= \lim_{x\to\infty} \frac{2+\frac{3}{x}}{\sqrt{3-\frac{1}{x}-\frac{2}{x^2}}} = \frac{2}{\sqrt{3}}$$

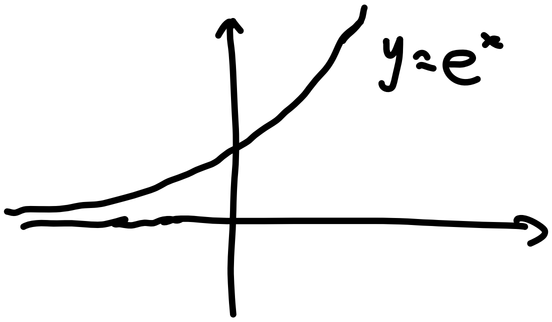
(4) $$\lim_{x\to\infty} \frac{x^2+3}{\sqrt{2x^4-x+1}+x^2}$$

$$= \lim_{x\to\infty} \frac{1+\frac{3}{x^2}}{\sqrt{\frac{1}{x^4}}\sqrt{2x^4-x+1}+1}$$

$$= \lim_{x\to\infty} \frac{1+\frac{3}{x^2}}{\sqrt{2-\frac{1}{x^3}+\frac{1}{x^4}}+1} = \frac{1}{\sqrt{2}+1}$$

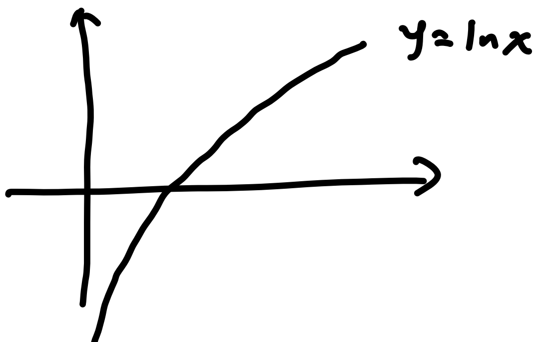
(5) $\lim_{x\to-\infty} e^{\frac{x^2+1}{x+1}} = 0$

$$\lim_{x\to-\infty} \frac{x^2+1}{x+1} = \lim_{x\to-\infty} \frac{x+\frac{1}{x}}{1+\frac{1}{x}} = -\infty$$



(6) $\lim_{x\to\infty} \ln \frac{x-1}{3x^2+2} = -\infty$

$$\lim_{x\to\infty} \frac{x-1}{3x^2+2} = \lim_{x\to\infty} \frac{1-\frac{1}{x}}{3x+\frac{2}{x}} = 0$$



$$\lim_{x\to4} \frac{x-4}{\sqrt{x}-2}$$

(7) $\lim_{x\to\infty} \frac{x-\sqrt{x^2+x}}{1}$

$$= \lim_{x\to\infty} \frac{(x-\sqrt{x^2+x})(x+\sqrt{x^2+x})}{x+\sqrt{x^2+x}}$$

$$= \lim_{x\to\infty} \frac{x^2-(x^2+x)}{x+\sqrt{x^2+x}}$$

$$= \lim_{x\to\infty} \frac{-x}{x+\sqrt{x^2+x}}$$

$$= \lim_{x\to\infty} \frac{-1}{1+\sqrt{\frac{1}{x^2}}\sqrt{x^2+x}}$$

$$= \lim_{x\to\infty} \frac{-1}{1+\sqrt{1+\frac{1}{x}}} = -\frac{1}{2}$$

2. $$\lim_{x\to\infty} \frac{e^x-e^{-x}}{e^x+e^{-x}} = \lim_{x\to\infty} \frac{1-e^{-2x}}{1+e^{-2x}} = 1$$

$$\lim_{x\to-\infty} \frac{e^x-e^{-x}}{e^x+e^{-x}} = \lim_{x\to-\infty} \frac{e^{2x}-1}{e^{2x}+1} = -1$$

$\Rightarrow$ H.A. at $y=1$, $y=-1$

3. $f(x) = \frac{1}{x-1} - \frac{1}{x}$ Domain : $x \neq 1, 0$

$$\lim_{x \to 1^-} f(x) = -\infty, \quad \lim_{x \to 1^+} f(x) = \infty$$

$$\lim_{x \to 0^-} f(x) = \infty, \quad \lim_{x \to 0^+} f(x) = -\infty$$

$$\lim_{x \to \infty} f(x) = 0 \qquad \lim_{x \to -\infty} f(x) = 0$$

$$f'(x) = -\frac{1}{(x-1)^2} + \frac{1}{x^2} = \frac{-x^2 + (x-1)^2}{(x-1)^2 x^2}$$

$$= \frac{-x^2 + (x^2 - 2x + 1)}{(x-1)^2 x^2} = \frac{-(2x-1)}{(x-1)^2 x^2}$$

$$f'(x) = 0 \Rightarrow x = \frac{1}{2}$$

f'	$+$		$+$		$-$		$-$
f	$\nearrow$	0	$\nearrow$	$\frac{1}{2}$	$\searrow$	1	$\searrow$

$x^3 > (x-1)^3$ for any x

$$f''(x) = 2 \cdot \frac{1}{(x-1)^3} - 2\frac{1}{x^3} = 2 \cdot \frac{x^3 - (x-1)^3}{(x-1)^3 x^3}$$

$x^3 - (x-1)^3 > 0$

$\Rightarrow f'(x) = 0$ no root

f''	$+$		$-$		$+$
f	$\cup$	0	$\cap$	1	$\cup$

$f(\frac{1}{2}) = -4$