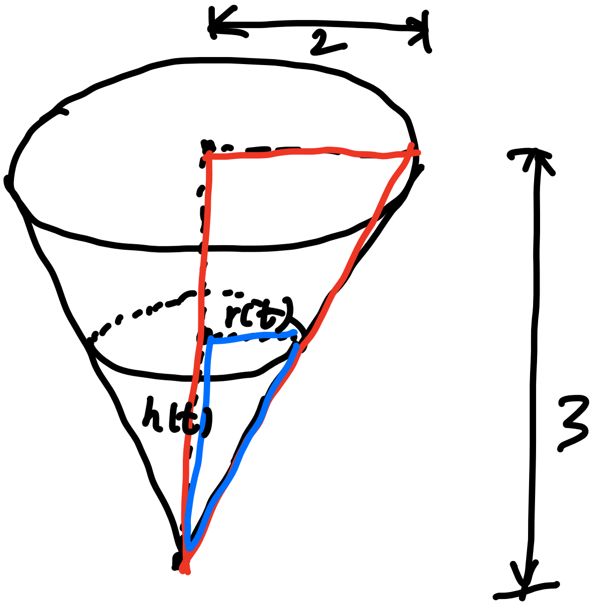
1.



$$\frac{h(t)}{r(t)} = \frac{3}{2} \qquad h(t) = \frac{3}{2} r(t)$$

$V(t)$ $\qquad$ $A(t)$

$$V(t) = \frac{1}{3}\pi r(t)^2 h(t) = \frac{1}{3}\pi r(t)^2 \cdot \frac{3}{2} r(t) = \frac{\pi}{2} r(t)^3$$

$$A(t) = \pi r(t)^2$$

$$V'(t) = \frac{\pi}{2} 3 r(t)^2 r'(t)$$

$$A'(t) = \pi \cdot 2 r(t) r'(t)$$

At t_0, $V'(t_0) = 10$, $A(t_0) = 1$, $A'(t_0) = ?$

$\hookrightarrow 1 = \pi r(t_0)^2$

$r(t_0) = \frac{1}{\sqrt{\pi}}$

$$10 = \frac{3\pi}{2} \cdot \left(\frac{1}{\sqrt{\pi}}\right)^2 r'(t_0) \Rightarrow r'(t_0) = \frac{20}{3}$$

$$A'(t_0) = 2\pi \cdot \frac{1}{\sqrt{\pi}} \cdot \frac{20}{3} = \frac{40}{3}\sqrt{\pi}.$$

2. $f(x) = x^4 - x^2$

Domain: $(-\infty, \infty)$

$f'(x) = 4x^3 - 2x = 2x(2x^2 - 1)$

$f'(x) = 0 \Rightarrow x = 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

f'	$-$		$+$		$-$		$+$
f	$\searrow$	$-\frac{1}{\sqrt{2}}$	$\nearrow$	0	$\searrow$	$\frac{1}{\sqrt{2}}$	$\nearrow$

$f(0) = 0$

$f\left(\frac{1}{\sqrt{2}}\right) = f\left(-\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

$f''(x) = 12x^2 - 2 = 2(6x^2 - 1)$

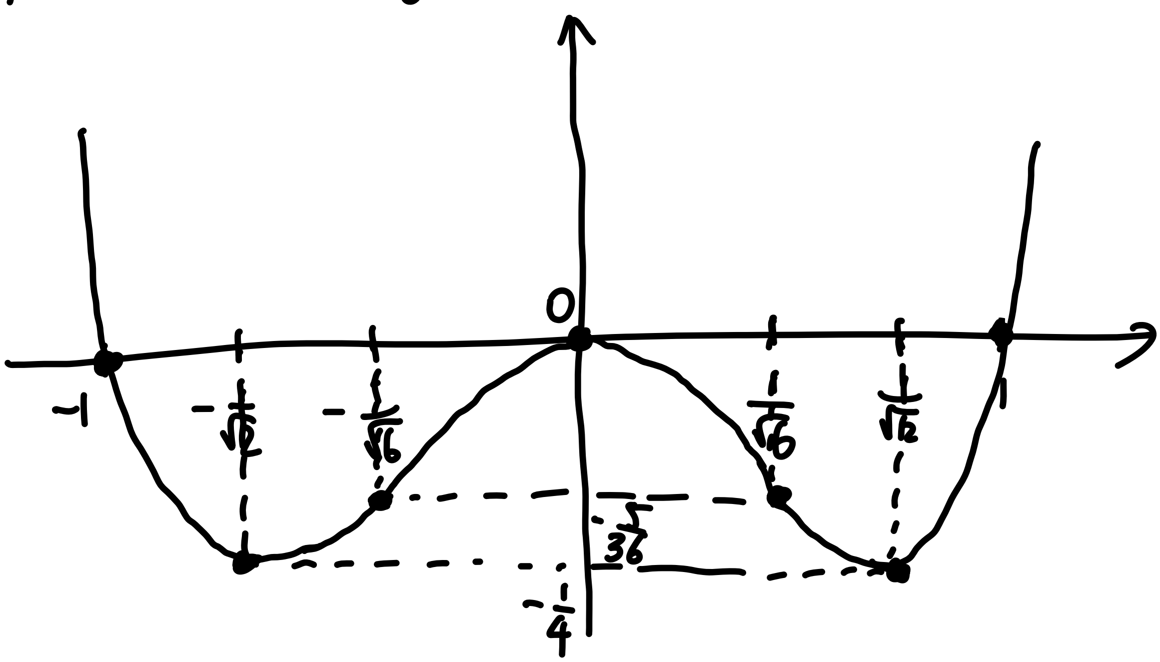
$f''(x) = 0 \Rightarrow x = \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}$

f''	$+$		$-$		$+$
f	$\cup$	$-\frac{1}{\sqrt{6}}$	$\cap$	$\frac{1}{\sqrt{6}}$	$\cup$

$f\left(\frac{1}{\sqrt{6}}\right) = f\left(-\frac{1}{\sqrt{6}}\right)$

$= \frac{1}{36} - \frac{1}{6} = -\frac{5}{36}$

$f(1) = f(-1) = 0$



3. $f(x) = \dfrac{x^2}{x-1}$

Domain : $x \neq 1$

$\lim\limits_{x \to 1^-} f(x) = -\infty$. $\lim\limits_{x \to 1^+} f(x) = \infty$.

$$f'(x) = \frac{2x \cdot (x-1) - x^2 \cdot 1}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2}$$

$$= \frac{x(x-2)}{(x-1)^2}$$

$f'(x) = 0 \Rightarrow x = 0, 2$

$$\begin{array}{c|ccccccc} f' & + & & - & & - & & + \\ \hline & & 0 & & 1 & & 2 & \\ f & \nearrow & & \searrow & & \searrow & & \nearrow \end{array}$$

$f(0) = 0$

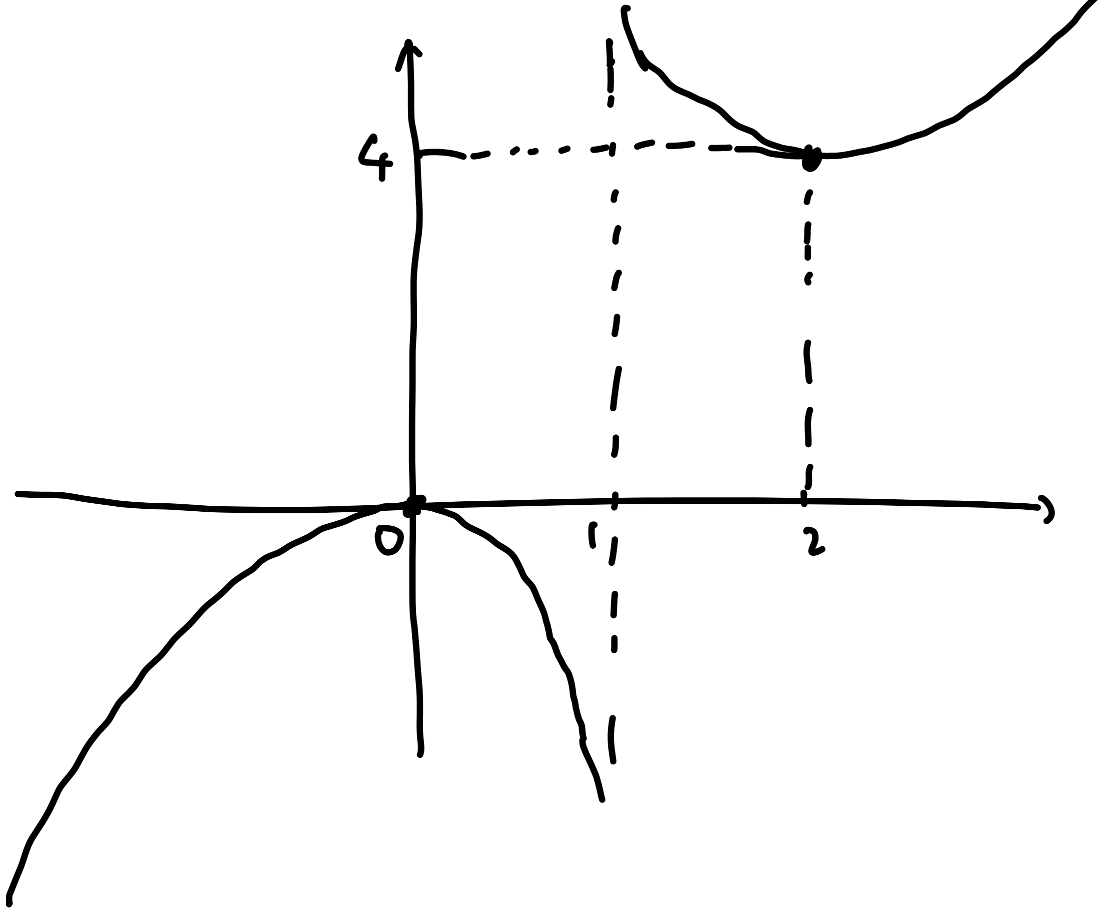
$f(2) = 4$

$$f''(x) = \frac{(2x-2)(x-1)^{\cancel{2}} - (x^2 - 2x) \cdot 2\cancel{(x-1)}}{(x-1)^{\cancel{4}3}}$$

$$= \frac{2x^2 - 4x + 2 - 2x^2 + 4x}{(x-1)^3} = \frac{2}{(x-1)^3}$$

$f''(x) = 0 \Rightarrow$ none

$$\begin{array}{c|ccc} f'' & - & & + \\ \hline & & 1 & \\ f & \cap & & \cup \end{array}$$


4
0
1
2