1. $f(x) = x^{1/3}$ $\qquad f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$

$L(x) = f(a) + f'(a)(x-a)$ $\qquad a = 8$

$f(8) = 8^{1/3} = 2$

$f'(8) = \frac{1}{3} 8^{-\frac{2}{3}} = \frac{1}{12}$

$\Rightarrow L(x) = 2 + \frac{1}{12}(x-8)$

$8.01^{1/3} \approx L(8.01) = 2 + \frac{1}{12} \times 0.01 \approx 2.00083$

2. $V = x^3$ $\qquad V' = 3x^2$

$dV = 3x^2 dx$

At $x = 4$, $\quad -0.02 \le dx \le 0.02$

$\Delta V \approx dV = 3 \times 4^2 dx = 48\, dx$

$\Rightarrow -0.96 \le \Delta V \le 0.96$

(measured value of V is $64 \pm 0.96\ m^3$)

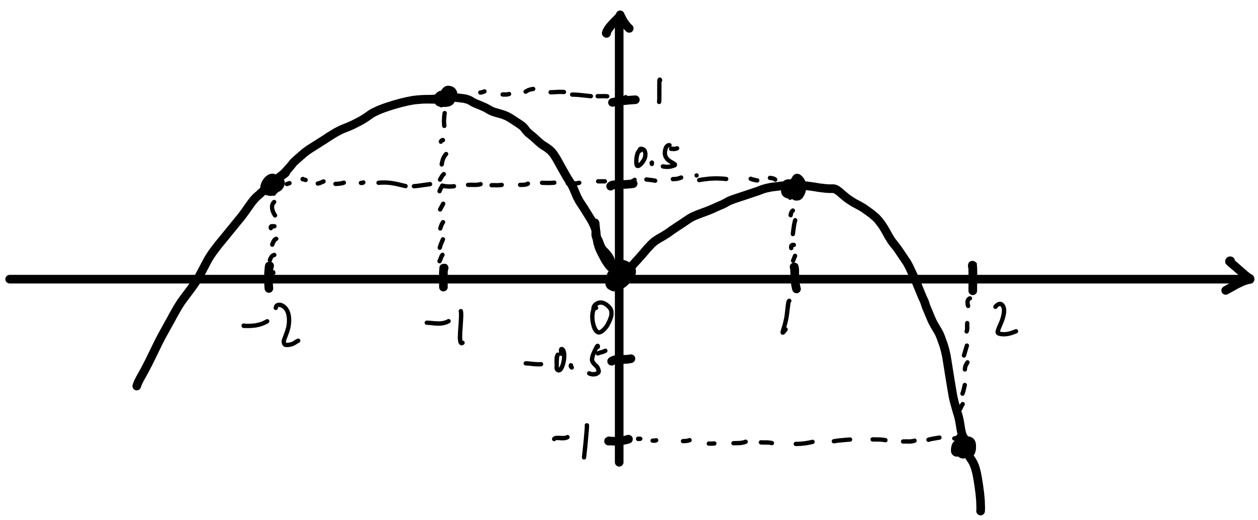
- Relative error (percentage error) of a quantity q is $\frac{\Delta q}{q}$

In Problem 2, the relative error for the side length is $\frac{0.02}{4} = 0.5\%$

The relative error for the volume is

$\frac{0.96}{64} = 1.5\%$

3.



(1) local max: @ -1, 1

local min: @ 0

(2) On $(-2, 2)$, global max @ $f(-1) = 1$

global min DNE.

4. Extra question: Find all local max and min.

$f(x) = x^3 - 2x^2 + x + 1$

$f'(x) = 3x^2 - 4x + 1$

$f'(x) = 0 \Rightarrow (3x-1)(x-1) = 0$

$x = \frac{1}{3}, 1$ both are in $[0,2]$

$$f\left(\frac{1}{3}\right) = \frac{1}{27} - \frac{2}{9} + \frac{1}{3} + 1 = \frac{31}{27}$$

$$f(1) = 1 - 2 + 1 + 1 = 1$$

$$f(0) = 1$$

global min.

$$f(2) = 8 - 2 \times 4 + 2 + 1 = 3 \longrightarrow \text{global max}$$

$$f''(x) = 6x - 4$$

$$f''\left(\frac{1}{3}\right) = 6 \times \frac{1}{3} - 4 = -2 < 0 \Rightarrow \text{local max @ } \frac{1}{3}$$

$$f''(1) = 6 \times 1 - 4 = 2 > 0 \Rightarrow \text{local min @ } 1.$$