

Total time: 15 minutes.

Problem 1 (4 points). Find the derivative of the following function by definition.

$$f(x) = x - \frac{1}{x}$$

Take any $x \neq 0$.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h) - \frac{1}{x+h} - (x - \frac{1}{x})}{h} = 1 + \lim_{h \rightarrow 0} \frac{-\frac{1}{x+h} + \frac{1}{x}}{h} \\ &= 1 + \lim_{h \rightarrow 0} \frac{(-\frac{1}{x+h} + \frac{1}{x})x(x+h)}{hx(x+h)} = 1 + \lim_{h \rightarrow 0} \frac{-x + (x+h)}{hx(x+h)} = 1 + \lim_{h \rightarrow 0} \frac{1}{x(x+h)} = 1 + \frac{1}{x^2} \end{aligned}$$

Problem 2 (2 points each). Calculate derivatives.

1. $\frac{d}{dt}(2t^2 - 3t + \frac{1}{\sqrt{t}}) = 4t - 3 - \frac{1}{2}t^{-3/2}$
2. $\frac{d}{dx}(x^2 \cos x + \frac{2}{x}) = 2x \cos x - x^2 \sin x - \frac{2}{x^2}$
3. $((x^2 - 2x) \sin x)' = (2x - 2) \sin x + (x^2 - 2x) \cos x$