Math 2250 — Quiz 2 (20220906) Solution

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**Total time: 15 minutes.**

**Notice**: When determining limits, calculate the value of the limit if it exists; otherwise, if it is positive/negative infinity, please indicate; otherwise, please write DNE.

**Problem 1 (6 points).** Find the points of discontinuity of the following function:

$$f(x) = \begin{cases} \sin x + 1 & , \text{ when } x \le 0 \\ x^2 & , \text{ when } 0 < x \le 2 \\ \frac{x^2-4}{x-2} & , \text{ when } x > 2 \end{cases}$$

We only need to check the continuity at $x = 0, 2$ since $f$ is clearly continuous elsewhere.

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (\sin x + 1) = 1, \quad \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0$$

Therefore $f$ is discontinuous at 0.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2 = 4, \quad \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x^2-4}{x-2} = 4, \quad f(2) = 2^2 = 4$$

Therefore $f$ is continuous at 4.
In conclusion, $f$ is discontinuous at 0.

**Problem 2 (4 points).** Determine the following limit.

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x^2-9x}$$

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x^2-9x} = \lim_{x\to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{x(x-9)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{x-9}{x(x-9)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{1}{x(\sqrt{x}+3)} = \frac{1}{54}$$