MATH 2250 QUIZ 1 (20220830) SOLUTION
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Total time: 15 minutes.

Notice: When determining limits, calculate the value of the limit if it exists; otherwise, if it is positive/negative infinity, please indicate; otherwise, please write DNE.

Problem 1 (2 points each). Determine the following limits.

$$\lim_{x\to 1} \frac{1-x}{x^2+2x-3} \qquad \lim_{x\to 2^-} \frac{x^2+2x}{|x-2|} \qquad \lim_{x\to 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x}$$

$$\lim_{x\to 1} \frac{1-x}{x^2+2x-3} = \lim_{x\to 1} \frac{1-x}{(x+3)(x-1)} = \lim_{x\to 1} \frac{-1}{x+3} = -\frac{1}{4}$$

$$\lim_{x\to 2^-} \frac{x^2+2x}{|x-2|}$$

The numerator approaches 8, and the denominator approaches 0 while being positive. Therefore the limit is $+\infty$.

$$\lim_{x\to 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x} = \lim_{x\to 0} \frac{2(2+x)(\frac{1}{2+x}-\frac{1}{2})}{2(2+x)x} = \lim_{x\to 0} \frac{2-(2+x)}{2(2+x)x} = \lim_{x\to 0} \frac{-1}{2(2+x)} = -\frac{1}{4}$$

Problem 2 (4 points). Find all vertical asymptotes of the following function.

$$f(x) = \frac{|2x+2|}{x^2-1}$$

$$f(x) = \frac{|2x+2|}{x^2-1} = \frac{|2(x+1)|}{(x+1)(x-1)} = 2\cdot\frac{|x+1|}{x+1}\cdot\frac{1}{x-1}$$

We need to check the points $x=-1,1$.
At $x=-1$, $2\cdot\frac{1}{x-1}$ approaches -1, and $\lim_{x\to -1^-}\frac{|x+1|}{x+1}=-1$, $\lim_{x\to -1^+}\frac{|x+1|}{x+1}=1$. Therefore $\lim_{x\to -1^-} f(x)$ and $\lim_{x\to -1^+} f(x)$ are real numbers. Therefore $f(x)$ does not have VA at $x=-1$.
At $x=1$, $2\cdot\frac{|x+1|}{x+1}$ approaches 2, and $\lim_{x\to 1^-}\frac{1}{x-1}=-\infty$. Therefore $\lim_{x\to 1^-} f(x)=-\infty$ and $f(x)$ has VA at $x=1$.
As conclusion, $f(x)$ has VA at $x=1$.