Midterm 1 · average = 83

$$(\sin^{-1}x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$(\cos^{-1}x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\tan^{-1}x)' = \frac{1}{\sec^2(\tan^{-1}x)} = \frac{1}{1+x^2}$$

$f(x) = \tan x$ $\quad f^{-1}(x) = \tan^{-1}x$

$f'(x) = \sec^2 x$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \frac{1}{\cos^2 x} = \sec^2 x$

$\sec^2(\boxed{\tan^{-1}x}) = 1+x^2$

$\tan \square = x$ $\quad \sec^2 \square = 1 + x^2$

Ex $(x^2 \sin^{-1}x)' = 2x \sin^{-1}x + x^2 \cdot \frac{1}{\sqrt{1-x^2}}$

$$(\cos^{-1}(\boxed{e^{-2x}}))' = -\frac{1}{\sqrt{1-(e^{-2x})^2}} \cdot (e^{\boxed{-2x}})'$$

$$= -\frac{1}{\sqrt{1-(e^{-2x})^2}} \cdot e^{-2x} \cdot (-2)$$

$$\left(\tan^{-1}\left(\frac{\ln x}{e^x}\right)\right)' = \frac{1}{1+\left(\frac{\ln x}{e^x}\right)^2}\cdot\frac{\frac{1}{x}e^x - \ln x\cdot e^x}{e^{2x}}$$

4.1 Related rates

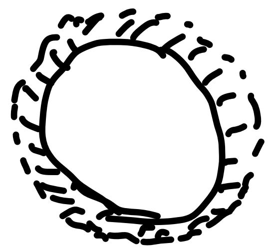
1. radius $r(t)$, volume $V(t)$ ← ①

$$V(t) = \frac{4}{3}\pi\left(r(t)\right)^3 \quad \leftarrow ②$$

take $\frac{d}{dt}$

$$V'(t) = \frac{4}{3}\pi\cdot 3\left(r(t)\right)^2\cdot r'(t) \quad \leftarrow ③$$

At given time t_0, we know



$r(t_0) = 3 \qquad r'(t_0) = 2$

$$\Rightarrow V'(t_0) = \frac{4}{3}\pi\cdot 3\cdot 3^2\cdot 2 = \boxed{72\pi}$$

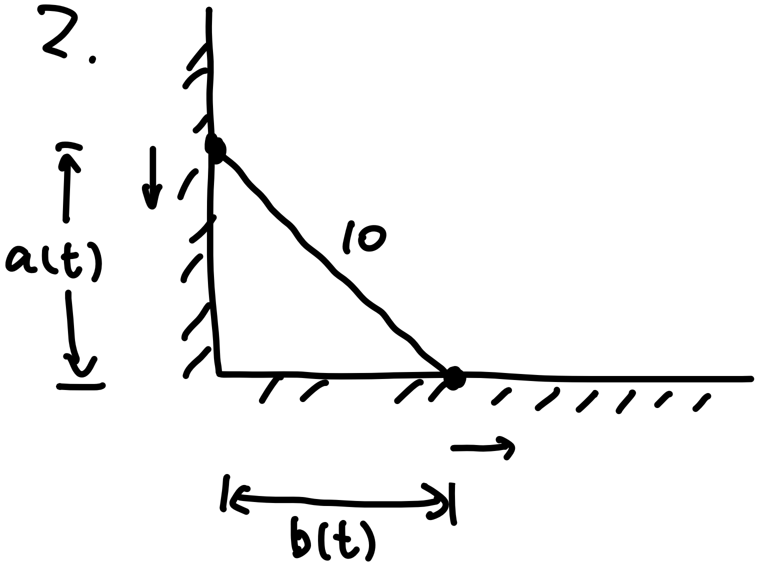
① Recognize relevant quantities (functions of t)

② Find relations between them

③ Take derivative

④ Plug in known info (at time t_0), solve (often you need to go back to original relation)

2.



$$(a(t))^2 + (b(t))^2 = 10^2$$

$$\Big\downarrow \frac{d}{dt}$$

$$2a(t)\cdot a'(t) + 2b(t)\cdot b'(t) = 0$$

At t_0, $a(t_0) = 6$, $a'(t_0) = -2$

want: $b'(t_0)$

$$6^2 + (b(t_0))^2 = 10^2$$

$$(b(t_0))^2 = 10^2 - 6^2 = 64$$

$$b(t_0) = 8$$

$$\Rightarrow 2\cdot 6\cdot(-2) + 2\cdot 8\cdot b'(t_0) = 0$$

$$16\, b'(t_0) = 24$$

$$b'(t_0) = \frac{24}{16} = \frac{3}{2}$$

3. surface area $S(t)$, radius $r(t)$

$$S(t) = 4\pi (r(t))^2$$

$$\Big\downarrow \frac{d}{dt}$$

$$S'(t) = 4\pi\cdot 2r(t)\cdot r'(t)$$

At t_0, $S(t_0) = 2$, $S'(t_0) = 5$

$\hookrightarrow \quad 2 = 4\pi \left(r(t_0)\right)^2$

$$\left(r(t_0)\right)^2 = \frac{1}{2\pi}$$

$$r(t_0) = \sqrt{\frac{1}{2\pi}}$$

$$\Rightarrow \quad 5 = 4\pi \cdot 2 \cdot \sqrt{\frac{1}{2\pi}} \cdot r'(t_0)$$

$$r'(t_0) = \frac{5}{8\pi \cdot \sqrt{\frac{1}{2\pi}}}.$$