

1. (1)

$$\left(\frac{x^2+1}{x^2-1} + \frac{x^2}{x+2} \right)'$$

$$= \frac{2x(x^2-1) - (x^2+1) \cdot 2x}{(x^2-1)^2} + \frac{2x \cdot (x+2) - x^2}{(x+2)^2}$$

(2)

$$\left(\frac{x \tan x}{1 + \cos x} \right)'$$

$$= \frac{(x \tan x)' \cdot (1 + \cos x) - x \tan x \cdot (-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{(\tan x + x \sec^2 x)(1 + \cos x) + x \tan x \sin x}{(1 + \cos x)^2}$$

(3)

$$\frac{d}{dt} \left(\frac{t + \sin t}{t^2 \cos t} \right)$$

$$= \frac{(1 + \cos t) t^2 \cos t - (t + \sin t)(2t \cos t - t^2 \sin t)}{t^4 \cos^2 t}$$

(4)

$$(x^2 \cos x + x^3)'$$

$$= 2x \cos x - x^2 \sin x + 3x^2$$

$$(x^2 \cos x + x^3)'$$

$$= 2 \cos x - 4x \sin x - x^2 \cos x + 6x$$

$$(5) (x^5 + \sin x)^{(29)}$$

$$29 = 28 + 1$$

$$= (x^5)^{(29)} + (\sin x)^{(29)}$$

$$= \cos x$$

$$2. f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$$

$$\underset{h=1}{\uparrow} \approx \frac{f(4) - f(3)}{1} = \frac{23 - 13}{1} = 10$$

Alternatively, take $h = -1$

$$f'(3) \approx \frac{f(2) - f(3)}{-1} = \frac{6 - 13}{-1} = 7$$

$$3. \underline{f(a+h)} \approx \underline{f(a)} + f'(a)h \quad \text{for small } h$$

$$\begin{array}{c} \uparrow \\ \sqrt{3.999} \end{array} \quad \begin{array}{c} \uparrow \\ \sqrt{4} \end{array}$$

$$f(x) = \sqrt{x}$$

$$a = 4$$

$$h = -0.001$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$\begin{aligned}
 f(3.999) &\approx f(4) + f'(4) \cdot (-0.001) \\
 &= 2 + \frac{1}{2\sqrt{4}} \cdot (-0.001) \\
 &= 1.99975
 \end{aligned}$$

$$4. \quad s(t) = \frac{1}{t+1} + \frac{t}{4} + k, \quad t \geq 0$$

$$(1) \quad s(0) = 0 \quad \frac{1}{0+1} + \frac{0}{4} + k = 0 \Rightarrow k = -1$$

$$(2) \quad v(t) = \frac{0 \cdot (t+1) - 1 \cdot 1}{(t+1)^2} + \frac{1}{4} = -\frac{1}{(t+1)^2} + \frac{1}{4}$$

$$a(t) = -\frac{0 \cdot (t+1) - 1 \cdot (2t+2)}{(t+1)^4} = \frac{2t+2}{(t+1)^4} = \frac{2}{(t+1)^3}$$

$$(3) \quad v(t) = 0 \Rightarrow -\frac{1}{(t+1)^2} + \frac{1}{4} = 0$$

$$\Rightarrow t = 1 \quad \text{or} \quad \cancel{2}$$

$$(4) \quad v(t) > 0 \quad \begin{array}{c} v(t) \quad - \quad + \\ | \quad \quad | \quad \quad \rightarrow \\ 0 \quad \quad 1 \end{array} \Rightarrow (1, \infty)$$