

**Problem 1.** Let  $W = \text{Span} \left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \right\}$ . Compute the orthogonal projection of  $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$  onto  $W$

We first get an orthogonal basis for  $W$  (by Gram-Schmidt):

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\mathbf{u}_2 = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} - \frac{3+0+2}{1+0+4} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

Then

$$\text{proj}_W \mathbf{v} = \frac{1+0+0}{1+0+4} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \frac{2-4+0}{4+4+1} \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} \\ 0 \\ \frac{2}{5} \end{pmatrix} + \begin{pmatrix} -\frac{4}{9} \\ \frac{4}{9} \\ \frac{2}{9} \end{pmatrix} = \begin{pmatrix} -\frac{11}{45} \\ \frac{4}{45} \\ \frac{28}{45} \end{pmatrix}$$