**Problem 1.** Let $A = \begin{pmatrix} 2 & 1 \\ -9 & 2 \end{pmatrix}$. Find a real invertible matrix P and a real matrix C of the for $C = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ such that $A = PCP^{-1}$.

$$\det(A - \lambda I) = (2-\lambda)^2 - 1 \cdot (-9) = (\lambda - (2+3i))(\lambda - (2-3i))$$

Take $\lambda_1 = a - bi = 2 - 3i$, $a = 2$, $b = 3$. To find complex eigenvectors,

$$\left(\begin{array}{cc|c} 3i & 1 & 0 \\ -9 & 3i & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 3i & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

Therefore one eigenvector is $\mathbf{v}_1 = \begin{pmatrix} 1 \\ -3i \end{pmatrix}$, and its real and imaginary parts are $Re\,\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $Im\,\mathbf{v}_1 = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$. Therefore we have $A = PCP^{-1}$ with

$$P = \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}$$

(the correct final answer may be non-unique)