MATH 461 — QUIZ 6 SOLUTION

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Problem 1. Let P_2 be the vector space of all polynomials with degree at most 2.
(1) (7 points) Show that $\{p(x) \in P_2 : p(3) = 0\}$ is a subspace.
(2) (3 points) Show that $\{p(x) \in P_2 : p(3) = 1\}$ is not a subspace.

(1) Denote $V = \{p(x) \in P_2 : p(3) = 0\}$.
(i) $0 \in V$ because the zero polynomial, evaluated at 3, is equal to 0.
(ii) Closed under addition: Let $p(x), q(x) \in V$. Then $p(3) = 0, q(3) = 0$. Then $(p+q)(3) = p(3) + q(3) = 0$. Therefore $(p+q)(x) \in V$.
(iii) Closed under scalar multiplication: Let $p(x) \in V$, $c \in \mathbb{R}$. Then $p(3) = 0$. Then $(cp)(3) = c \cdot p(3) = 0$. Therefore $(cp)(x) \in V$.

(2) $0 \notin \{p(x) \in P_2 : p(3) = 1\}$ because the zero polynomial, evaluated at 3, is equal to 0, not equal to 1.