

Problem 1. Compute

$$\det \begin{pmatrix} 0 & 1 & 2 & -1 \\ 1 & -1 & 2 & 1 \\ 3 & 1 & 0 & 4 \\ -1 & 2 & 0 & 2 \end{pmatrix}$$

Use row operations

$$\begin{pmatrix} 0 & 1 & 2 & -1 \\ 1 & -1 & 2 & 1 \\ 3 & 1 & 0 & 4 \\ -1 & 2 & 0 & 2 \end{pmatrix}$$

(row exchanged)

$$\begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 2 & -1 \\ 3 & 1 & 0 & 4 \\ -1 & 2 & 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 4 & -6 & -2 \\ 0 & 1 & 2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -14 & 2 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

Therefore the determinant of the original matrix is

$$(-1) \cdot 1 \cdot 1 \cdot (-14) \cdot 4 = 56$$

(where the first -1 comes from the row exchange)