

**Problem 1.** Determine whether  $\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$  is in the span of  $\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$ ,  $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$  and  $\begin{pmatrix} -2 \\ 3 \\ -7 \end{pmatrix}$ .

It is equivalent to determine whether the linear system with the following augmented matrix has a solution.

$$\begin{pmatrix} 0 & 2 & -2 & 2 \\ 1 & -1 & 3 & -1 \\ -2 & 3 & -7 & -3 \end{pmatrix}$$

By row operations,

$$\begin{pmatrix} 1 & -1 & 3 & -1 \\ 0 & 2 & -2 & 2 \\ -2 & 3 & -7 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 3 & -1 \\ 0 & 2 & -2 & 2 \\ 0 & 1 & -1 & -5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 3 & -1 \\ 0 & 2 & -2 & 2 \\ 0 & 0 & 0 & -6 \end{pmatrix}$$

The last row gives a contradiction. Therefore this linear system has no solution, and therefore

$\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$  is not in the span of  $\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$ ,  $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$  and  $\begin{pmatrix} -2 \\ 3 \\ -7 \end{pmatrix}$ .