MATH 246 QUIZ 8 SOLUTION

INSTRUCTOR: RUIWEN SHU

Problem 1. Solve the initial value problem

$$y' + y = u\,(t-2)\,e^{-3t},\;\; y\,(0) = 0$$

using the Laplace transform, where u denotes the Heaviside function. Express your answer as a piecewise defined function.

Write $u(t-2)e^{-3t} = u(t-2)e^{-3(t-2)} \cdot e^{-6}$. Then taking Laplace transform gives

$$s\mathcal{L}[y] - y(0) + \mathcal{L}[y] = e^{-6} \cdot e^{-2s} \cdot \frac{1}{s+3}$$

$$\mathcal{L}[y] = e^{-6} \cdot e^{-2s} \cdot \frac{1}{(s+3)(s+1)} = e^{-6} \cdot e^{-2s} \cdot (\frac{1}{2} \cdot \frac{1}{s+1} - \frac{1}{2} \cdot \frac{1}{s+3})$$

by partial fractions. Then take inverse Laplace transform,

$$y(t) = e^{-6} \cdot e^{-2s} \cdot \frac{1}{(s+3)(s+1)} = e^{-6} \cdot \frac{1}{2}u(t-2)e^{-(t-2)} - e^{-6} \cdot \frac{1}{2}u(t-2)e^{-3(t-2)}$$

$u(t-2) = 1$ for $t \geq 2$ and $u(t-2) = 0$ for $0 \leq t < 2$. Therefore

$$y(t) = \begin{cases} 0, & 0 \leq t < 2 \\ e^{-6} \cdot \frac{1}{2}e^{-(t-2)} - e^{-6} \cdot \frac{1}{2}e^{-3(t-2)}, & t \geq 2 \end{cases}$$