MATH 246 QUIZ 7 SOLUTION
INSTRUCTOR: RUIWEN SHU

Problem 1. Compute the Laplace transform of $(t^2+1)e^t$.

$$\mathcal{L}[(t^2+1)e^t] = \mathcal{L}[t^2e^t] + \mathcal{L}[e^t] = \frac{2}{(s-1)^3} + \frac{1}{s-1}$$

where we used the formula $\mathcal{L}[t^k e^{ct}] = \frac{k!}{(s-c)^{k+1}}$.

Problem 2. Compute the inverse Laplace transform of $\frac{11-s}{s^2-2s-3}$.

First do partial fractions:

$$\frac{11-s}{s^2-2s-3} = \frac{11-s}{(s-3)(s+1)} = \frac{C_1}{s-3} + \frac{C_2}{s+1}$$

$$11-s = C_1(s+1) + C_2(s-3)$$

Evaluate at $s=3$ gives $11-3 = C_1(3+1)$, $C_1 = 2$. Evaluate at $s=-1$ gives $11-(-1) = C_2(-1-3)$, $C_2 = -3$. Therefore

$$\mathcal{L}^{-1}[frac11-ss^2-2s-3] = \mathcal{L}[\frac{2}{s-3}] + \mathcal{L}[\frac{-3}{s+1}] = 2e^{3t} - 3e^{-t}$$