MATH 246 — MIDTERM EXAM 3

19 NOV 2020, INSTRUCTOR: RUIWEN SHU

Total time: 75 minutes

Notice:
(1) Write your solution to each problem on a DIFFERENT answer sheet.
(2) Write your name on every answer sheet.
(3) You are allowed to use calculators, the textbook, and your notes in this exam.
(4) You are NOT allowed to use MATLAB or the internet in this exam.
(5) You do not need to write the interval of definition unless you are asked to do so.

Problem 1. (20=5+5+5+5 points) Compute: ($\mathcal{L}$ denotes the Laplace transform, $\mathcal{L}^{-1}$ denotes the inverse Laplace transform)

$$(1)\,\mathcal{L}[\cos(2t)];\quad (2)\,\mathcal{L}[t^2e^t(e^t+e^{2t})];\quad (3)\,\mathcal{L}^{-1}[\frac{s}{(s+1)(s+3)}];\quad (4)\,\mathcal{L}^{-1}[\frac{1}{s^3+s}];$$

Express your solution with real-valued functions.

Problem 2. (20 points) Solve the initial value problem

$$y' + 2y = u(t-1)e^{3t},\quad y(0) = 1$$

where u denotes the Heaviside function. Express your solution as a piecewise defined function (u should not appear in your final answer).

Problem 3. (20=10+10 points) Consider the ODE system

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = A(t)\mathbf{x},\quad A(t) = \begin{pmatrix} 2t+1 & -3 \\ 1 & 2t+5 \end{pmatrix}$$

(1) Find the constant c such that

$$\mathbf{x}_1(t) = e^{t^2+ct}\begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

is a solution.

(2) Given that

$$\mathbf{x}_2(t) = e^{t^2+4t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

is a solution. Is $\mathbf{x}_1, \mathbf{x}_2$ a fundamental set of solutions? Justify you answer.

More problems on the next page!

Problem 4. (25=15+10 points) Define the matrix

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$$

(1) Find e^{tA}.

(2) Solve the initial value problem

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = A\mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

Problem 5. (15 points) Find e^{tA} for the matrix

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$