MATH 246 GROUP WORK 10

6 NOV 2020, INSTRUCTOR: RUIWEN SHU

**Each group only needs to submit ONE file containing your solutions!**

**Problem 1.** Consider the linear system of ODEs

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

(1) Check that the following are a fundamental set of solutions:

$$\mathbf{x}_1 = e^t\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \mathbf{x}_2 = e^{2t}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

Check Liouville's Wronskian theorem for this set of solutions. What is the general solution?

(2) Solve the initial value problem with initial condition

$$\mathbf{x}(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

(Similar as before, you evaluate the general solution at $t = 0$, match with the initial condition, and solve for $C_1, C_2$.)

**Problem 2.** Consider the linear system of ODEs

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

(1) Notice that $x_2(t)$ satisfies an ODE by itself. Solve and get its general solution.

(2) Substitute your general solution of $x_2(t)$ into the ODE for $x_1(t)$, solve it, and obtain the general solution to the system.

(3) Write your general solution as $\mathbf{x}(t) = C_1\mathbf{x}_1(t) + C_2\mathbf{x}_2(t)$, and check that $\mathbf{x}_1(t), \mathbf{x}_2(t)$ are indeed a fundamental set of solutions.

(4) Solve the initial value problem with initial condition

$$\mathbf{x}(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$