MATH 246 — QUIZ 5 SOLUTION

INSTRUCTOR: RUIWEN SHU

Problem 1. Solve the initial-value problem

$$y'' - y' - 12y = 0, \quad y(0) = 0,\, y'(0) = -7$$

The characteristic polynomial is

$$p(z) = z^2 - z - 12 = (z-4)(z+3)$$

It has two roots $r_1 = 4$, $r_2 = -3$, both with multiplicity 1. Therefore the general solution is

$$y(t) = C_1 e^{4t} + C_2 e^{-3t}$$

Matching with the initial condition gives

$$0 = C_1 + C_2, \quad -7 = 4C_1 - 3C_2$$

Solve and get

$$C_1 = -1, \quad C_2 = 1$$

Therefore the solution is

$$y(t) = -e^{4t} + e^{-3t}$$

Problem 2. Solve the initial-value problem

$$y'' - 2y' + 5y = 0, \quad y(0) = 2,\, y'(0) = 0$$

The characteristic polynomial is

$$p(z) = z^2 - 2z + 5 = (z - (1+2i))(z - (1-2i))$$

It has two roots $r_1 = 1 + 2i$, $r_2 = 1 - 2i$, both with multiplicity 1. Therefore the general solution is (writing into real form)

$$y(t) = C_1 e^{(1+2i)t} + C_2 e^{(1-2i)t} = C_1 e^t(\cos 2t + i \sin 2t) + C_2 e^t(\cos 2t - i \sin 2t) = \tilde{C}_1 e^t \cos 2t + \tilde{C}_2 e^t \sin 2t$$

Then

$$y'(t) = (\tilde{C}_1 + 2\tilde{C}_2) e^t \cos 2t + (\tilde{C}_2 - 2\tilde{C}_1) e^t \sin 2t$$

Matching with the initial condition gives

$$2 = \tilde{C}_1, \quad 0 = \tilde{C}_1 + 2\tilde{C}_2$$

Solve and get

$$\tilde{C}_1 = 2, \quad \tilde{C}_2 = -1$$

Therefore the solution is

$$y(t) = 2e^t \cos 2t - e^t \sin 2t$$