

Problem 1. Find the interval of definition for the solution to the initial-value problem

$$(t^2 - 4)y'' + (\ln t)y + e^{2t}y = t + 2, \quad y(1) = 0, \quad y'(1) = 1$$

This is a linear equation, which can be written into the standard form as

$$y'' + \frac{\ln t + e^{2t}}{t^2 - 4}y = \frac{t + 2}{t^2 - 4}$$

The interval of definition for this IVP is the largest interval containing $t_I = 1$ such that all the coefficient functions are continuous. Bad points are $t = 2, -2$, and $\ln t$ requires $t > 0$. Therefore the interval of definition is $(0, 2)$.

Problem 2. Compute $Wr[t, t \ln t]$.

$$Wr[t, t \ln t] = \det \begin{pmatrix} t & t \ln t \\ (t)' & (t \ln t)' \end{pmatrix} = \det \begin{pmatrix} t & t \ln t \\ 1 & \ln t + 1 \end{pmatrix} = t(\ln t + 1) - t \ln t \cdot 1 = t$$