MATH 246 Midterm Exam 2

22 Oct 2020, Instructor: Ruiwen Shu

# Total time: 75 minutes

**Notice:**
**(1) Write your solution to each problem on a DIFFERENT answer sheet.**
**(2) Write your name on every answer sheet.**
**(3) You are allowed to use calculators, the textbook, and your notes in this exam.**
**(4) You are NOT allowed to use MATLAB or the internet in this exam.**
**(5) You do not need to write the interval of definition unless you are asked to do so.**

**Problem 1. (10 points)** Compute the interval of definition of the following initial value problem

$$t^2y'' + \frac{t}{t+2}y' + 3y + \ln(3-t) = 0, \quad y(1) = 1,\, y'(1) = -2$$

**Problem 2. (20 points)** Determine whether

$$Y_1(t) = t^2 + 2t, \quad Y_2(t) = 2t^2 - 3t + 1, \quad Y_3(t) = -t^2 + 12t - 2$$

are a fundamental set of solutions to the ODE

$$y''' = 0$$

Justify your answer.

**Problem 3. (30=20+10 points)**

(1) Find the Green's function $g$ associated with the operator

$$L = \frac{\mathrm{d}^2}{\mathrm{d}t^2} + 4\frac{\mathrm{d}}{\mathrm{d}t} + 13$$

(in other words, $L$ is defined by $Ly = y''+4y'+13y$.) Express your answer using real-valued functions.

(2) Use $g$ to compute a particular solution to

$$y'' + 4y' + 13y = e^{-2t}$$

## More problems on the next page!

**Problem 4. (20 points)** Find the general solution to the ODE

$$y'' + 3y' - 4y + e^t(e^t - 1) = 0$$

Express your answer using real-valued functions.

**Problem 5. (20 points)** Find the general solution to the ODE

$$L(t)y = y'' - \frac{2t+2}{t^2+2t+2}y' + \frac{2}{t^2+2t+2}y = t^2 + 2t + 2$$

given that a fundamental set of solutions to $L(t)y = 0$ is

$$Y_1(t) = t + 1, \quad Y_2(t) = t^2 + 2t$$