

Each group only needs to submit ONE file containing your solutions!

Problem 1. Solve the initial value problem by Laplace transform:

$$y'' + 2y' + 5y = 1, \quad y(0) = 1, \quad y'(0) = 0$$

(If you don't want complex numbers in the partial fractions, you might need to take the real/imaginary parts of the formula $\mathcal{L}[e^{(a+bi)t}] = \frac{1}{s-a-bi}$ to get new formulas involving trigonometric functions.)

Problem 2. Solve the initial value problem by Laplace transform:

$$y' + 2y = \begin{cases} \frac{1}{\epsilon}, & 0 \leq t < \epsilon \\ 0, & t \geq \epsilon \end{cases}, \quad y(0) = 0$$

where $\epsilon > 0$ is a constant.

Denote the solution you obtained as $y_\epsilon(t)$. As $\epsilon \rightarrow 0$, does $y_\epsilon(t)$ converge to some function $Y(t)$ (if you focus on $t \in [\epsilon, \infty)$)? Does $Y(t)$ solve any initial value problem related to this one?

Hint: recall the limit $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$.