MATH 246 Quiz 3 solution

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Problem 1. A population of termites grows at a rate proportional to its population and doubles every week. Assuming the population starts at 5000 termites, how many weeks will it take until the population reaches 25,000? (Round your answer to three decimal places.)

Denote the population as $p(t)$. It satisfies an exponential model, and therefore $p(t) = p_I e^{rt}$ where p_I is the initial population, and r is a constant. The first condition gives $p(1) = 2p_I$, which is $p_I e^r = 2p_I$. Therefore $e^r = 2$, $r = \ln 2$.
If the population starts from $p_I = 5000$ and it takes time t to reach population 25000, then $p(t) = 5000e^{\ln 2 \cdot t} = 25000$, $e^{\ln 2 \cdot t} = 5$, $t = \frac{\ln 5}{\ln 2} \approx 2.322$.

Problem 2. For the initial value problem $y' = t^2 - 2y$, $y(1) = 2$, use Euler's method with a step size of $h = 0.05$ to estimate the value of $y(1.1)$. Round your answer to three decimal places.

The initial time $t_0 = 1$. Then $t_1 = 1.05$, $t_2 = 1.1$, t_2 is the final time.

$$y_0 = 2$$

$$y_1 = y_0 + hf(t_0, y_0) = 2 + 0.05(1^2 - 2 \cdot 2) = 1.85$$

$$y_2 = y_1 + hf(t_1, y_1) = 1.85 + 0.05(1.05^2 - 2 \cdot 1.85) \approx 1.720$$

The approximation of $y(1.1)$ is 1.720.