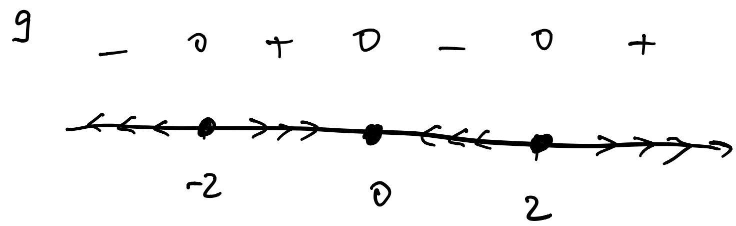

Figure 1: Phase portrait.

MATH 246
INSTRUCTOR: RUIWEN SHU

QUIZ 2 SOLUTION

Problem 1. Solve the initial-value problem

$$xy^2y' = 2\ln x, \quad y(1) = -1.$$

This is a separable ODE.

$$y^2\,\mathrm{d}y = \frac{2\ln x}{x}\,\mathrm{d}x$$

Integrate, (use a substitution $u = \ln x$ on the right hand side)

$$\frac{1}{3}y^3 + C = (\ln x)^2$$

Determine C by plugging in $x = 1$, $y = -1$:

$$-\frac{1}{3} + C = 0, \quad C = \frac{1}{3}$$

Solve for y:

$$y^3 = 3(\ln x)^2 - 1, \quad y = (3(\ln x)^2 - 1)^{1/3}$$

Problem 2. For the differential equation $y' = y^3 - 4y$, classify the stability of the stationary solution $y = 2$.

$$g(y) = y^3 - 4y = y(y+2)(y-2)$$

See page top for its phase portrait. The arrows are pointing away from $y = 2$ if started near it, and therefore $y = 2$ is unstable.