

Problem 1. Determine the interval of definition for the initial-value problem

$$y' + \frac{t+7}{t^2-t-2} = 0, \quad y(-2) = 0$$

This is an explicit ODE. The function $\frac{t+7}{t^2-t-2}$ is defined and continuous except when $t^2-t-2 = (t-2)(t+1) = 0$, that is, $t = 2$ or $t = -1$. The interval of definition for the initial-value problem is the largest interval containing the initial time -2 but not containing 2 or -1 . Therefore the answer is $(-\infty, -1)$.

Problem 2. Find the general solution to the differential equation

$$ty' - 2y + t^3 = 0$$

This is a linear equation by writing it as

$$y' - \frac{2}{t}y = -t^2$$

Then $a(t) = -\frac{2}{t}$, $A(t) = \int(-\frac{2}{t})dt = -2\ln|t|$, and the integrating factor is $e^{A(t)} = \frac{1}{t^2}$.

$$\left(\frac{1}{t^2}y\right)' = -1$$

$$\frac{1}{t^2}y = -t + C$$

$$y = -t^3 + Ct^2$$