Problem 1. Solve the initial value problem

$$y'' = t(y')^2, \quad y(0) = 3, \quad y'(0) = 2$$

Define $u = y'$, then the original ODE becomes

$$u' = (1+t)u^2$$

This is a first order ODE for $u(t)$, and is separable.

$$\frac{du}{u^2} = tdt$$

$$-\frac{1}{u} + C_1 = \frac{1}{2}t^2$$

The initial condition $y'(0) = 2$ gives $u(0) = 2$, and therefore

$$-\frac{1}{2} + C_1 = \frac{1}{2}0^2 \Rightarrow C_1 = \frac{1}{2}$$

Therefore

$$-\frac{1}{u} + \frac{1}{2} = \frac{1}{2}t^2$$

$$-\frac{1}{u} = \frac{1}{2}t^2 - \frac{1}{2}$$

$$u = -\frac{1}{\frac{1}{2}t^2 - \frac{1}{2}} = \frac{2}{1-t^2}$$

Since $y' = u = \frac{2}{1-t^2}$, we integrate to get

$$y = \int \frac{2}{1-t^2}dt = \int (\frac{1}{1+t} + \frac{1}{1-t})dt = \ln|1+t| - \ln|1-t| + C_2$$

Use the initial condition $y(0) = 3$,

$$3 = \ln|1+0| - \ln|1-0| + C_2 \Rightarrow C_2 = 3$$

Therefore

$$y = \ln|1+t| - \ln|1-t| + 3$$