

Total time: 75 minutes

Notice:

- (1) Write your solution to each problem on a **DIFFERENT** answer sheet.
- (2) Write your name on every answer sheet.
- (3) You are allowed to use calculators, the textbook, and your notes in this exam.
- (4) You are **NOT** allowed to use **MATLAB** or the internet in this exam.
- (5) You do not need to write the interval of definition unless you are asked to do so.

Problem 1. (15=10+5 points)

(1) (10 points) Identify all the linear differential equation(s) among the following, and explain why the others are not:

$$(a): \frac{dx}{dt} = t \sin x + 1, \quad (b): y'' = ty' + 3y, \quad (c): y' = y^2 + t^2, \quad (d): \partial_t u = \partial_{xx} u$$

(2) (5 points) Compute the interval of definition of the solution to the initial value problem

$$\frac{dy}{dt} = \frac{y}{t^2 - 1} + \frac{t}{t + 2}, \quad y(0.1) = 1$$

Problem 2. (20=10+10 points)

(1) (10 points) Solve the initial value problem

$$t^2 y y' = y^2 + 1, \quad y(2) = 1$$

Express your solution in an explicit form.

(2) (10 points) Use the explicit Euler method to approximate $y(2.6)$ of the above initial value problem, with step size $h = 0.2$. Keep at least 3 digits after the decimal point, in all your intermediate results.

Problem 3. (20 points) Find the general solution to the second order ODE

$$x'' + 2x' + t = 0$$

More problems on the next page!

Problem 4. (20 points) Find the general solution to the ODE

$$(2y + x^5) dx + (y^3 + 2x) dy = 0$$

You may leave your final answer as an implicit function.

Problem 5. (25=15+5+5 points) Initially a tank contains 10 liters of water. At some instant, salt solution of concentration 2g/L starts to flow into the tank at a rate of 3L/h, while well-stirred mixture is flowing out, so that the volume of liquid in the tank stays the same.

(1) (15 points) Compute the concentration of salt in the tank $C(t)$.

(2) (5 points) Will the concentration of salt in the tank reach 1g/L? If yes, compute the first time this happens. If no, explain your answer.

(3) (5 points) Will the concentration of salt in the tank reach 3g/L? If yes, compute the first time this happens. If no, explain your answer.