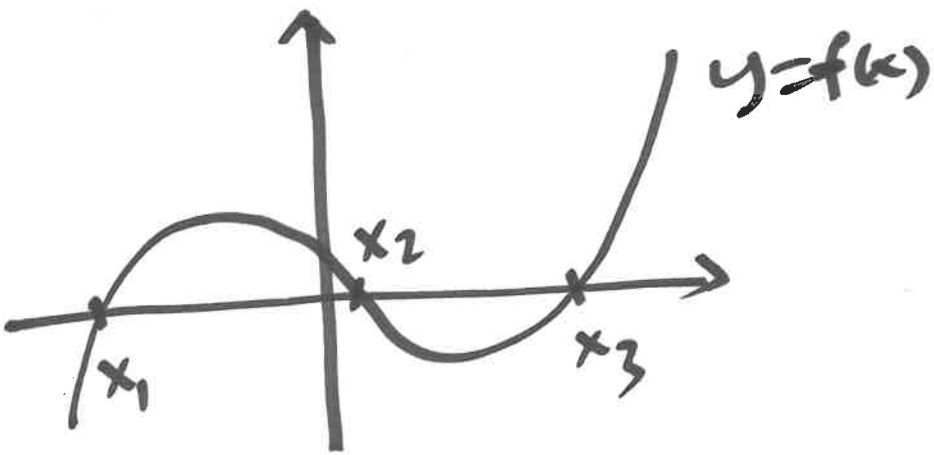
AMSC/CMSC 460 (numerical methods)

Contents in this course

Chap. 2 : Solution to equation in one variable

$f(x) = 0 \Rightarrow x = ?$

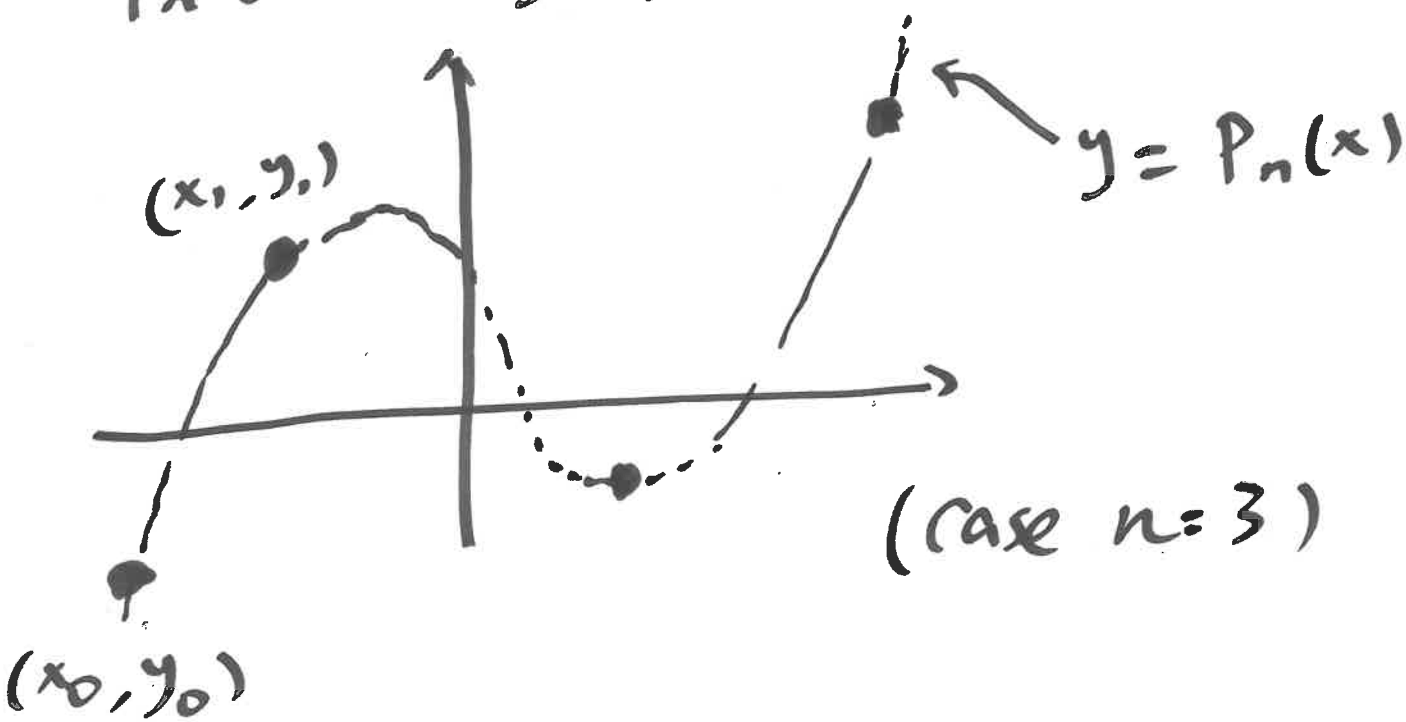


Chap 3, 8 : interpolation, polynomial approximation

- Given (x_i, y_i), $i = 0, 1, \cdots, n$, find

$\deg \le n$ polynomial $P_n(x)$ s.t.

$$P_n(x_i) = y_i, \quad i = 0, 1, \cdots, n$$

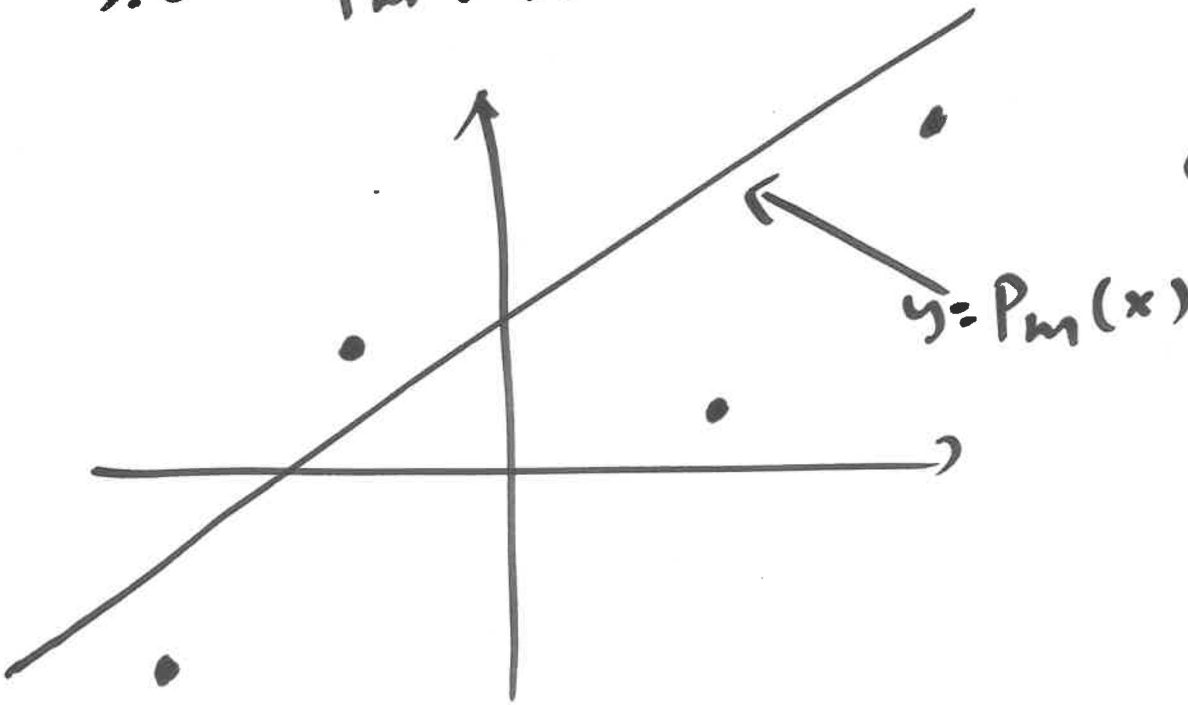


- Why polynomial approximation?

Weierstrass' theorem:
any continuous function on $[a, b]$ can be approximated by polynomials.

- For smooth functions, polynomial approximation is very good.

• Given (x_i, y_i), $i = 0, 1, \cdots, n$, find polynomial $P_m(x)$ w/ $\deg \le m$ $(m \le n)$ s.t. $P_m(x_i)$ is close to y_i, $i = 0, 1, \cdots, n$



case $n = 3$, $m = 1$

Chap 6: Direct solvers for linear systems

$$Ax = b$$

know A, b

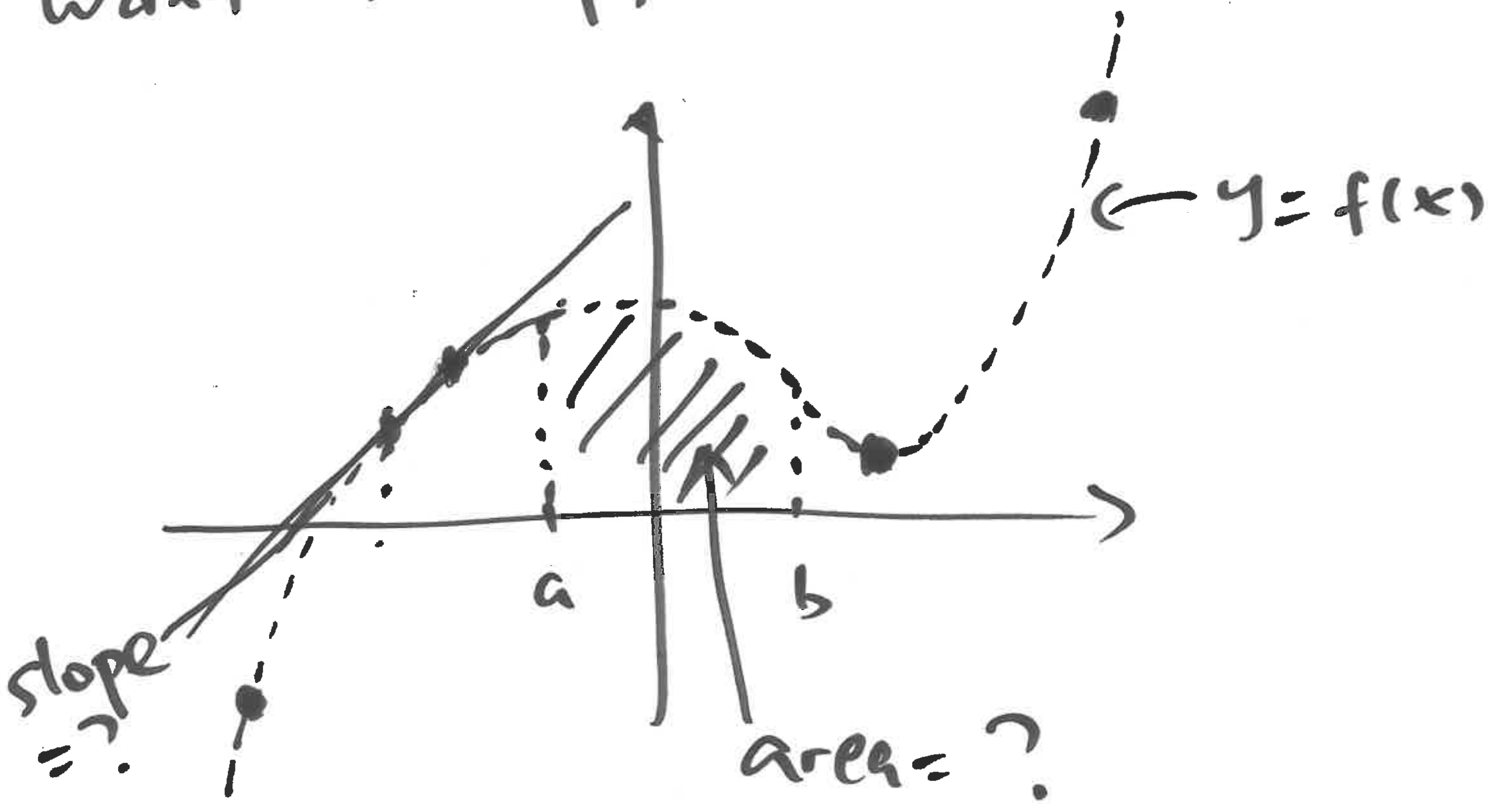
want x

$$\underset{n\times n}{\begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}} \underset{n\times 1}{\begin{pmatrix} \quad \\ \\ \\ \end{pmatrix}} = \underset{n\times 1}{\begin{pmatrix} \quad \\ \\ \\ \end{pmatrix}}$$

Chap 4: numerical differentiation/integration

Given point values $f(x_0), f(x_1), \ldots, f(x_n)$

want to approximate: $f'(x)$, $\int_a^b f(x)\,dx$



Chap 5: ODE solvers.

(initial value problems)

$$\frac{dy}{dt} = f(y,t), \quad y(t_0) = y_0$$

want $y(t)$

1.2 Round-off errors

Binary machine number (floating-point number)

$$\underset{\substack{+/- \\ s}}{0} \quad \underbrace{1001\cdots\cdots01}_{11 \text{ digits}} \quad \underbrace{0110 \cdots\cdots 0101}_{52 \text{ digits}}$$

s $\quad c: 0 \sim 2047 \quad f: 0 \sim 2^{52}-1$

represent $\rightarrow$ $\underbrace{(-1)^s}_{\text{sign}} \cdot \underbrace{2^{c-1023}}_{\text{scale}} \underbrace{(1+f)}_{\text{effective digits.}}$

Decimal machine number

$$\pm\, 0.d_1 d_2 \cdots d_k \times 10^n$$

$1 \le d_1 \le 9$, $0 \le d_i \le 9$, $i = 2, \cdots, k$

(k effective digits).

For real number $y = 0.d_1 d_2 \cdots d_k d_{k+1} \cdots \times 10^n$

the floating-point form $fl(y)$ is

- by chopping: $fl(y) = 0.d_1 \cdots d_k \times 10^n$
- by rounding: add $5 \times 10^{n-(k+1)}$ then chopping.

$\underline{\text{Ex}}$ Let $y = \frac{8}{7}$. Determine $fl(y)$ w/ 4 digits w/ chopping and rounding.

Check $fl(7y) \neq fl(7 fl(y))$

$$y = \frac{8}{7} = 0.11428\ldots \times 10^1$$

Chopping:

$$fl(y) = 0.1142 \times 10^1$$

$$7 fl(y) = 0.7994 \times 10^1$$

$$fl(7 fl(y)) = 0.7994 \times 10^1$$

$$fl(7y) = fl(0.8 \times 10^1) = 0.8 \times 10^1$$

rounding :

$$fl(y) = 0.1143 \times 10^1$$

$$7\,fl(y) = 0.8001 \times 10^1$$

$$fl(7\,fl(y)) = 0.8001 \times 10^1$$

$$fl(7y) = 0.8 \times 10^1$$