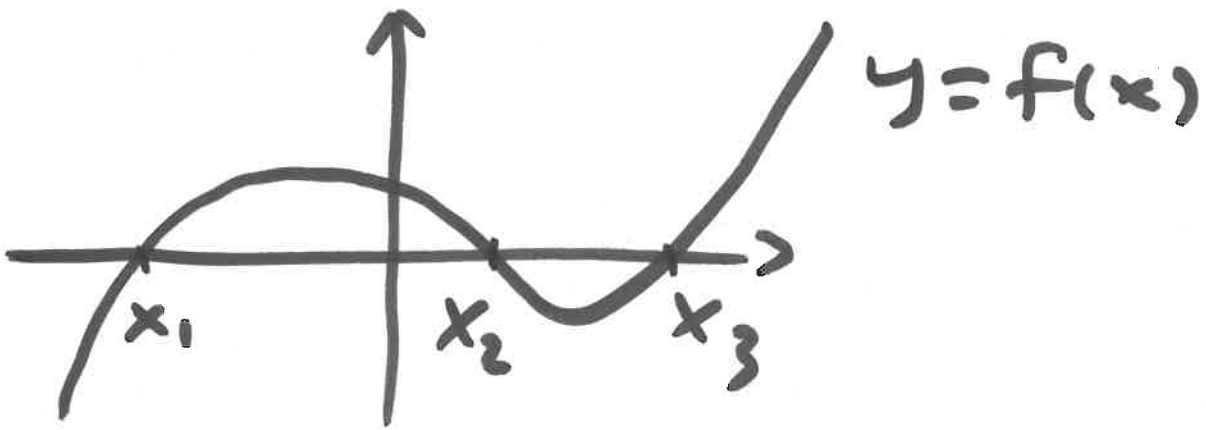
AMSC/CMSC 460

Contents in this Course

Chap 2 : Solution of equation in one variable

$f(x) = 0 \quad \Rightarrow \quad x = ?$



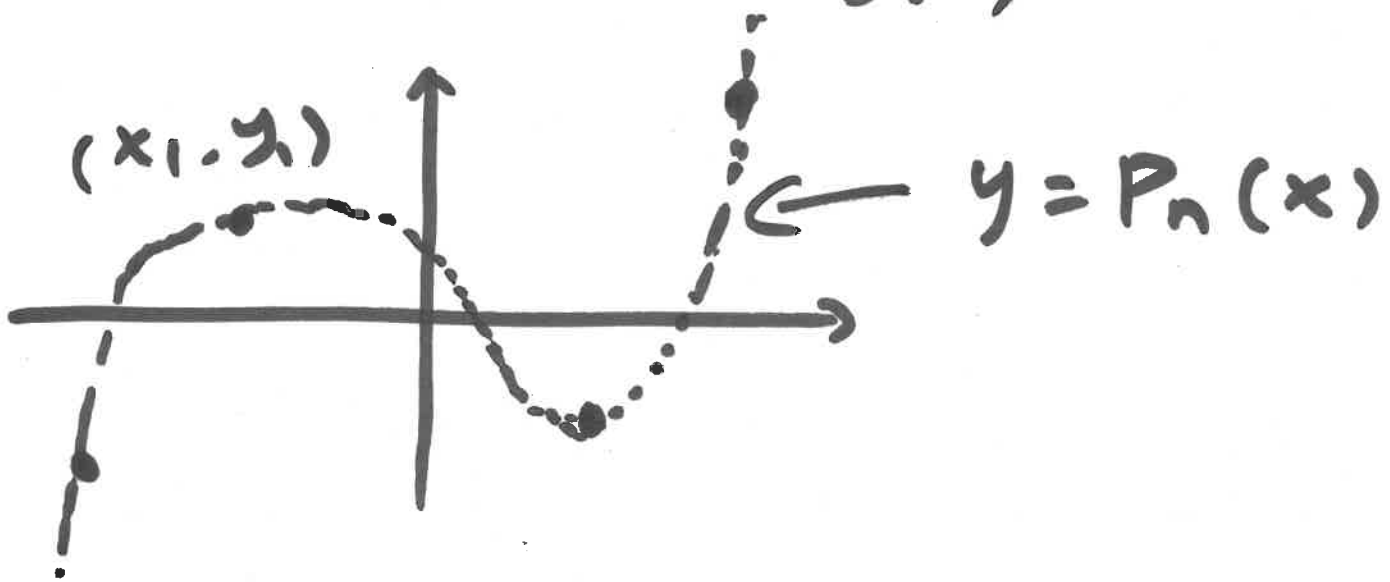
chap 3.8: interpolation.

polynomial approximation.

Given (x_i, y_i), $i = 0, 1, \cdots, n$

Find degree-n polynomial $P_n(x)$

s.t. $P_n(x_i) = y_i$, $i = 0, 1, \cdots, n$



- Weierstrass' theorem
- Smooth functions can be well-approximated by polynomials.

Chap 6 : Direct solvers for linear systems

$$Ax = b$$

know A, b

want x

$$\underset{n\times n}{\begin{pmatrix} & \\ & \end{pmatrix}} \underset{n\times 1}{\begin{pmatrix} \\ \end{pmatrix}} = \underset{n\times 1}{\begin{pmatrix} \\ \end{pmatrix}}$$

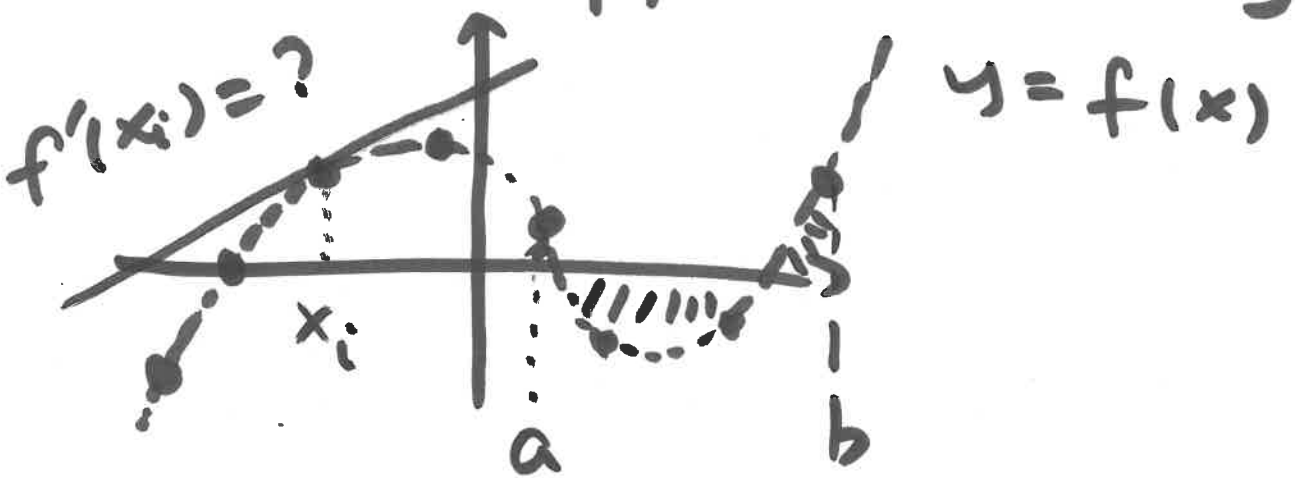
Chap 4 : Numerical differentiation and integration.

Given some point values

$f(x_0), \ldots, f(x_n)$

how to approximate $f'(x_i)$?

how to approximate $\int_a^b f(x)\,dx$?



Chap 5 : ODE solvers

initial value problems

$$\frac{dy}{dt} = f(y,t), \quad y(t_0) = y_0$$

want $y(t)$.

Other important things:

- Multi-D root finding: Chap 10
  related to big data
- Iterative linear system solvers
  often best for sparse matrices Chap 7
- Eigenvalue problems: Chap 9

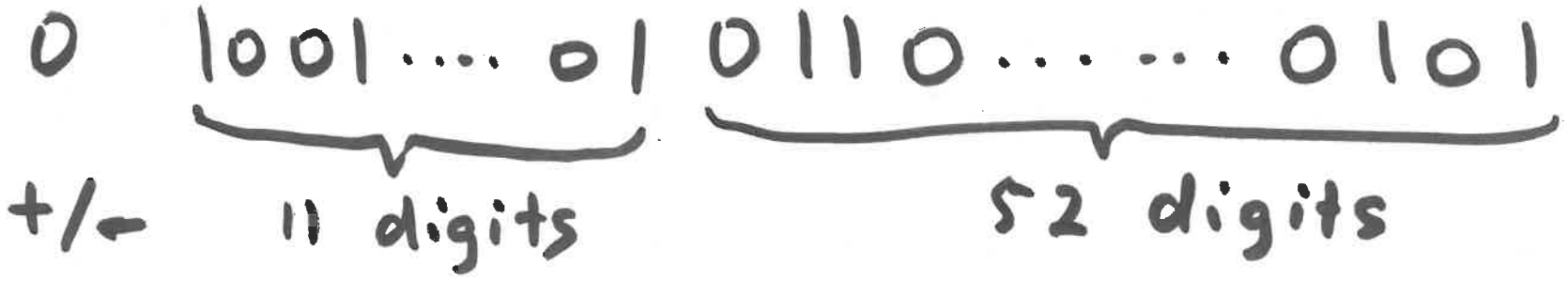
$$Ax = \lambda x \qquad \det(\lambda I - A) = 0$$

- Numerical PDE

1.2 Round-off errors

Binary machine number

(floating-point number)



s $\quad c: 0 \sim 2047 \quad f: 0 \sim 2^{52}-1$

$$\longrightarrow (-1)^s \, 2^{c-1023} \, (1+f)$$

sign scale effective digits

Decimal machine number

$$\pm 0.d_1 d_2 \cdots d_k \times 10^n$$

$1 \le d_1 \le 9$, $0 \le d_i \le 9$, $i = 2, \cdots, k$

For real number

$$y = 0.d_1 d_2 \cdots d_k d_{k+1} \cdots \times 10^n$$

the <u>floating-point form</u> $fl(y)$ is

- by chopping: $fl(y) = 0.d_1 \cdots d_k \times 10^n$
- by rounding: add $5 \times 10^{n-(k+1)}$ then chopping

<u>Ex</u> Let $y = \frac{8}{7}$. Determine $fl(y)$ with 4 digits, with chopping or rounding. Check $fl(7y) \neq fl(7fl(y))$

$$y = \frac{8}{7} = 1.1428\cdots = 0.11428\cdots \times 10^1$$

- chopping : $fl(y) = 0.1142 \times 10^1$

  $fl(7\, fl(y)) = 0.7994 \times 10^1$

- rounding : $fl(y) = 0.1143 \times 10^1$

  $fl(7\, fl(y)) = 0.8001 \times 10^1$