Math 136 mid-term Exam 1

Show all your work to get full credit!!!

Match 7, 2019

Problem 1. (1) (10pts) Find $\lim\limits_{x\to\infty} \dfrac{\sqrt{x+2019}}{\sqrt{2x+3}}$.

$$\lim_{x\to\infty} \frac{\sqrt{x+2019}}{\sqrt{2x+3}} = \lim_{x\to\infty} \frac{\sqrt{1+\frac{2019}{x}}}{\sqrt{2+\frac{3}{x}}} = \lim_{x\to\infty} \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

(2) (10pts) Find $\lim\limits_{x\to\infty} \dfrac{e^x-2}{e^{3x}+1}$.

$$\lim_{x\to\infty} \frac{e^x-2}{e^{3x}+1} = \lim_{x\to\infty} \frac{1-\frac{2}{e^x}}{e^{2x}+\frac{1}{e^x}} = 0$$

(3) (20pts) Find the value a, such that the function

$$f(x) = \begin{cases} \frac{1}{x}\left(\frac{3}{3+x}-1\right) & \text{if } x \neq 0 \\ a & \text{if } x = 0 \end{cases}$$

is continuous.

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1}{x}\left(\frac{3}{3+x}-1\right) = \lim_{x\to 0} \frac{1}{x}\frac{-x}{3+x} = \lim_{x\to 0} \frac{-1}{3+x} = -\frac{1}{3}$$

Therefore $a = -\frac{1}{3}$.

Problem 2. (20pts) Find the value $f'(1)$ if $f(x) = \dfrac{\ln(x^3+1)}{x}$.

$$f'(x) = \frac{\frac{3x^2}{x^3+1} \cdot x - \ln(x^3+1) \cdot 1}{x^2}$$
$$f'(1) = \frac{3}{2} - \ln 2$$

Problem 3. (20pts) Find the equation of the tangent line at $x = 0$ for the function $f(x) = e^{\cos x} \sin x$.

$$f'(x) = e^{\cos x}(-\sin x)\sin x + e^{\cos x} \cos x$$
$$f(0) = 0, \quad f'(0) = e$$

Tangent line:

$$y = ex$$

Problem 4. (20pts) Researchers have found a correlation between respiratory rate and body mass in the first 3 years of life. This correlation can be expressed by the function

$$\ln R(w) = 2.7 - 0.5 \ln w,$$

where w is body weight (in kg) and $R(w)$ is the respiratory rate (in breaths per minute).

(1) Find $R'(w)$ by first solving the equation for $R(w)$.

$$R(w) = e^{2.7 - 0.5 \ln w}$$
$$R'(w) = e^{2.7 - 0.5 \ln w}(-0.5\frac{1}{w})$$

(2) Find $R'(w)$ by using implicit differentiation.

$$\frac{1}{R(w)}R'(w) = -0.5\frac{1}{w}$$

$$R'(w) = -0.5\frac{1}{w}R(w)$$