Density: $f(X)$

Prob. $P(a \le X \le b) = \int_a^b f(x)dx$

mean: $\int_{-\infty}^{\infty} x f(x)\, dx = \mu$.

median: $\int_a^{\infty} f(x)\, dx = \frac{1}{2}$, $a = ?$

variance: $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx = ?$

deviation: $\sigma \geq 0$.

$$P(Z \geq 20) = \int_{20}^{60} f(x)\,dx$$

$$f(x) = \frac{1}{60},\ 0 \leq x \leq 60$$

$$= \int_{20}^{60} \frac{1}{60}\,dx$$

$$= \frac{40}{60} = \frac{2}{3}$$

$$\mu = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \cdot dx.$$

Rmk: $\int e^{x^2} dx$, $\int e^{-x^2} dx$.

Not able to evaluate $\Rightarrow \int_a^b e^{-x^2} dx$.

We can evaluate $\int_{-\infty}^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.

R.V. ξ $f(x) \leftarrow$ density fct.

Prob. $P(a \le \xi \le b) = \int_a^b f(x)\,dx$.

mean value: $\mu = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$. $\leftarrow$ Average

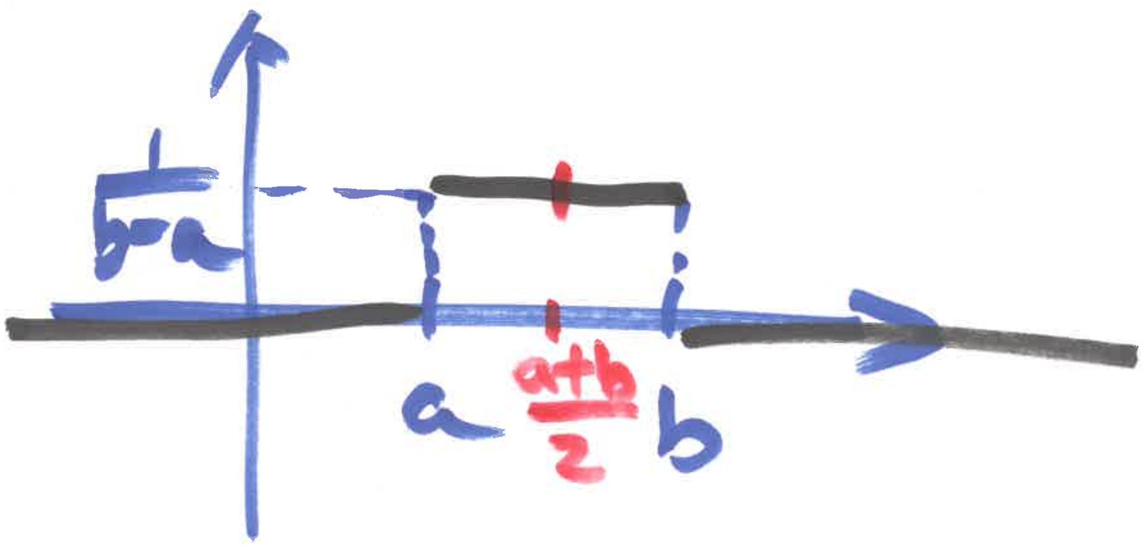
median value: a: $\int_a^{\infty} f(x)\,dx = \frac{1}{2}$
$= \int_{-\infty}^{a} f(x)\,dx$.

this is b/c. $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

Uniform distribution on [a, b]

$$f(x) = \begin{cases} 0, & x < a \\ \frac{1}{b-a}, & a \le x \le b \\ 0, & x > b \end{cases}$$

<u>mean</u> value / median value: $\frac{a+b}{2}$.



normal distribution / Gaussian distr.

density fct:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

(σ)

μ : mean value
σ : deviation
σ^2 : variance

μ : $\mu = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

σ^2 : $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)\, dx$

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

$$\underline{\cancel{*}}\ \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx$$

<u>Waiting time model:</u>

<u>density fct:</u>

$$f(t) = \begin{cases} 0 & \text{if } t<0 \\ a\cdot e^{-bt}, & \text{if } t\geq 0 \end{cases} \qquad \underline{a,b>0}$$

4.
$$\int_{-\infty}^{\infty} f(t)\,dt = 1.$$

$e^{\lambda t} \to \frac{1}{\lambda} e^{\lambda t}_{tc}$

$$1 = \int_{-\infty}^{\infty} f(t)\,dt = \int_{0}^{\infty} f(t)\,dt.$$

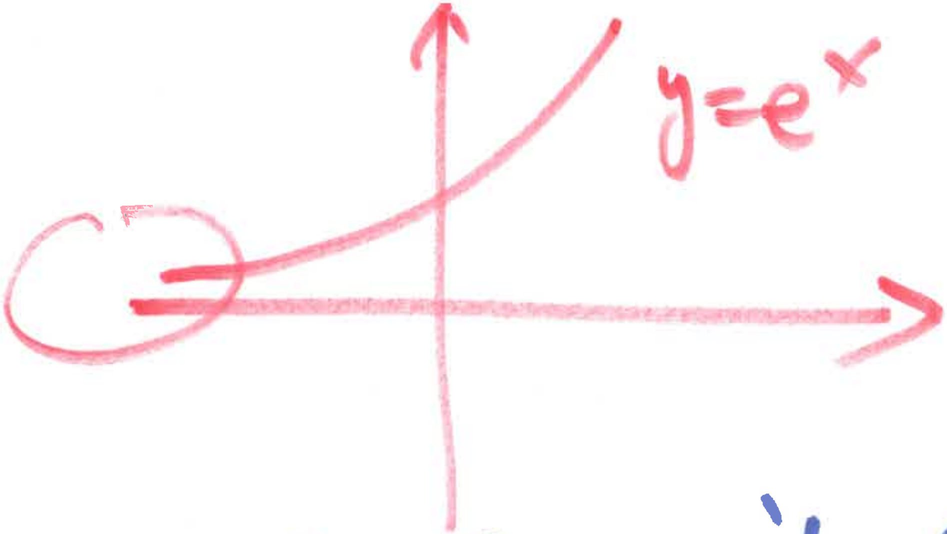
$$= \int_0^\infty a \cdot e^{-bt}\,dt = a\int_0^\infty e^{-bt}\,dt$$

$$= a \cdot \left(\frac{1}{-b} e^{-bt} \Big|_0^\infty \right)$$

$\boxed{a=b}$

$$\underline{\parallel} \quad = a \cdot \left(\frac{1}{-b} e^{-b \cdot \infty} - \frac{1}{-b} \cdot \overset{1}{\overset{\parallel}{e^0}} \right)$$

$$1 = a \cdot \left(\frac{1}{-b} \cdot \overset{\to 0}{e^{-\infty}} - \frac{1}{-b} \right) = \frac{a}{b}$$



$$f(t)=\begin{cases} 0 & \text{if } t<0 \\ ae^{-at} & \text{if } t\geq 0 \end{cases}$$

② <u>mean value</u> : $\boxed{\mu = \frac{1}{a}}$

$$\mu = \int_{-\infty}^{\infty} t\, f(t)\, dt = \int_0^{\infty} t\cdot f(t)\, dt.$$

$$= \int_0^{\infty} t\cdot a\cdot e^{-at}\, dt$$

$$\mu = a\int_0^\infty v\cdot u'\,dt \quad \boxed{\text{ІЗР}} \quad v(t)=t,\ v'(t)=1.$$

$$\begin{cases} u'(t) = a\cdot e^{-at} \\ u(t) = -e^{-at} \end{cases}$$

$$\mu = \left(v\cdot u\Big|_0^\infty - \int_0^\infty v'\cdot u\,dt\right)$$

$$\mu = \left(t\cdot(-e^{-at})\Big|_0^\infty\right) - \int_0^\infty 1\cdot(-e^{-at})\,dt.$$

$$= \underbrace{\left(\infty\cdot(-e^{-\infty}) - 0\right)}_{=0} + \int_0^\infty e^{-at}\,dt$$

$$\lim_{t\to\infty}\frac{t}{e^t} = 0$$

$$= 0 + \frac{1}{-a}e^{-at}\Big|_0^\infty$$

$$\mu = \underbrace{\frac{1}{-a}e^{-\infty}}_{=0} - \frac{1}{-a}\underbrace{e^{0}}_{=1} = \frac{1}{a}$$

$$f(t) = \begin{cases} 0 & \text{if } t<0 \\ ae^{-at} & \text{if } t \geq 0 \end{cases}$$

exponential density funct.

mean value : $\mu = \frac{1}{a}$.

Prob. : $P(c \leq Z \leq b) = \int_c^b f(t)\,dt$.

Exercise :

median value : λ : $\int_\lambda^\infty f(t)\,dt = \frac{1}{2}$.

$$P(X \leq 600) = \int_{-\infty}^{600} f(x)\,dx.$$

$$= \int_{0}^{600} f(x)\,dx$$

$$= \int_{0}^{600} \frac{1}{300} e^{-\frac{1}{300}x}\,dx.$$

$$= \frac{1}{300} \int_{0}^{600} e^{-\frac{1}{300}x}\,dx$$

$$= \cdots\cdots \quad \square.$$

$$f(t) = \begin{cases} \frac{1}{60} & 0 \leq t \leq 60 \\ 0 & \text{otherwise} \end{cases}$$

$$P(Z \geq 20) = P(20 \leq Z \leq 60)$$

$$= \int_{20}^{60} f(t)\,dt$$

$$= \int_{20}^{60} \frac{1}{60}\,dt$$

$$= \frac{60-20}{60} = \frac{40}{60} = \frac{2}{3}.$$

(a) $\mu = \frac{1}{a} = \frac{1}{.6} = \frac{5}{3}$.

(b). $P(2 \le 8 \le 5) = \int_2^5 0.6\, e^{-0.6t}\, dt$

$= -e^{-0.6t} \Big|_2^5$

$= -e^{-3} - (-e^{-1.2})$

$= \underline{-e^{-3} + e^{-1.2}}$. $\square$.

(c). $P(8 \geq 3) = \int_3^{\infty} f(t)\,dt$

$= \int_3^{\infty} 0.6\, e^{-0.6t}\,dt$

$= -e^{-0.6t} \Big|_3^{\infty}$

$= \left(\underbrace{-e^{-0.6\cdot\infty}}_{0}\right) + \left(+e^{-1.8}\right)$

$= e^{1.8} = e^{-1.8}$