Random Variable.

$$\begin{Bmatrix} X(1) = \text{up } 0 \\ X(2) = 1 \\ X(3) = 0. \end{Bmatrix}$$

$0 \rightarrow$ up

$1 \rightarrow$ down.

---

<u>Random V.</u> $X(n) = \begin{cases} 0 \\ 1. \end{cases}$

<u>$\{0, 1\} \rightarrow$ range.</u>

Random Variable:

$\Rightarrow$ **Probability**: $\begin{cases} P(X=0)=\frac{1}{2} \\ P(X=1)=\frac{1}{2} \end{cases}$

$$P(a \le X \le b) = ?$$

**Density function**: $f(x) \geq 0$.

$\rightarrow$ $P(a \le X \le b) = \int_a^b f(x)\,dx$.

density fct: $f(x) \rightsquigarrow \underline{\int_{-\infty}^{\infty} f(x)\,dx = 1}$

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx.$$

$$P(-\infty < X \leq b) = \lim_{a \to -\infty} \int_a^b f(x)\,dx$$

$$= \int_{-\infty}^b f(x)\,dx$$

$$\lim_{b \to +\infty} P(-\infty < X < b) = \lim_{b \to +\infty} \int_{-\infty}^b f(x)\,dx =$$

$$1 = P(-\infty < X < \infty) = \int_{-\infty}^{\infty} f(x)\,dx = 1.$$

Distribution function. $\underline{\gamma(n) = C}$. $(-\infty, \infty)$

$\gamma(t) =$

$$\underline{F(x) = \int_{-\infty}^{x} f(s)\,ds}.$$

$$\underline{= \lim_{a \to -\infty} \int_{a}^{x} f(s)\,ds}.$$

Property: $\lim_{x \to \infty} F(x) = \lim_{x \to \infty} \int_{-\infty}^{x} f(x)\,dx$

$$\lim_{x \to -\infty} F(x) = 0. \Leftarrow \underline{\int_{a}^{a} f(x)\,dx = 0}.$$

$$= \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

---

$$\Rightarrow \lim_{x \to -\infty} F(x) = 0 \;,\; \lim_{x \to +\infty} F(x) = 1 .$$

---

**Relation:** $F(x) = \int_{-\infty}^{x} f(s)\,ds .$

$$\rightarrow \underline{F'(x) = f(x)} .$$

$$P(0.225 \leq X \leq 0.240) = \int_{0.225}^{0.240} f(x)\,dx$$

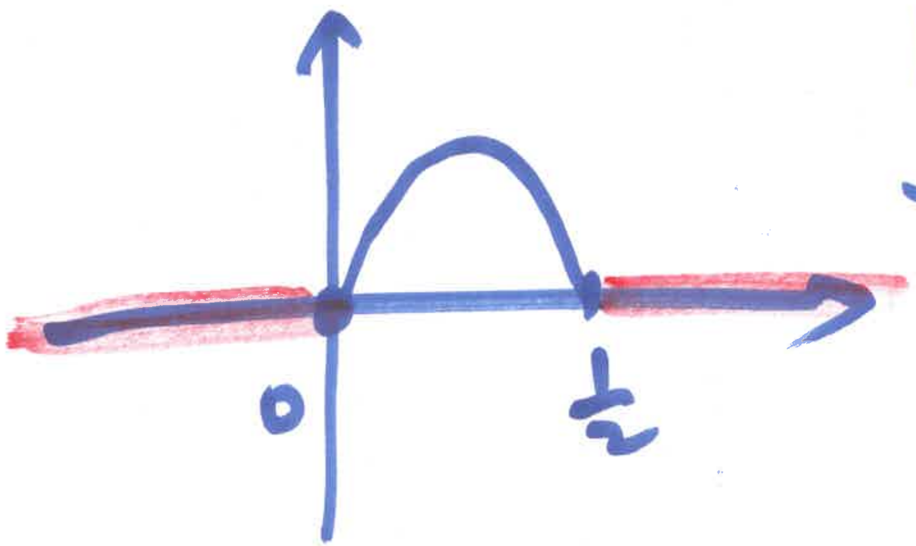
$$= \int_{0.225}^{0.240} 192\left(\frac{1}{2}-x\right)^2 \cdot x \cdot dx$$

$$= 192 \int_{0.225}^{0.240} \left(\frac{1}{4} - x + x^2\right) x\,dx$$

$$= 192 \int_{0.225}^{0.240} \left(\frac{x}{4} - x^2 + x^3\right) dx$$

$$\underline{0.0479} = 192\left[\frac{x^2}{8} - \frac{1}{3}x^3 + \frac{1}{4}x^4\right]_{0.225}^{0.240}$$

$$f(x) = \begin{cases} 192x\left(\frac{1}{2}-x\right)^2 & 0 \le x \le \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$




$$\int_{-\infty}^{\infty} f(x)\,dx$$

$$= \underbrace{\int_{-\infty}^{0} f(x)\,dx}_{=0} + \int_{0}^{\frac{1}{2}} f(x)\,dx$$

$$+ \underbrace{\int_{\frac{1}{2}}^{+\infty} f(x)\,dx}_{=0}$$

$$\int_0^{\frac{1}{2}} 192\, x\left(\tfrac{1}{2}-x\right)^2 dx$$

$$= 192\int_0^{\frac{1}{2}} x\left(\tfrac{1}{4}-x+x^2\right) dx$$

$$= 192\int_0^{1/2}\left(\frac{x}{4}-x^2+x^3\right) dx.$$

$$= 192\left[\frac{x^2}{8}-\frac{x^3}{3}+\frac{x^4}{4}\right]_0^{1/2}.$$

$$= 192\left[\frac{(\frac{1}{2})^2}{8}-\frac{(\frac{1}{2})^3}{3}+\frac{(\frac{1}{2})^4}{4}\right] = 1.$$

Uniform distribution: on $[a, b]$

density function equals to a constant.

$$f(x) = \begin{cases} 0, & x < a \\ \boxed{\frac{1}{b-a}}, & a \le x \le b \\ 0, & x > b. \end{cases} \Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$1 = \int_a^b f(x)\,dx = \int_a^b \alpha\,dx = \alpha x \Big|_a^b$$

$$= \alpha(b-a) = 1.$$

$$\alpha = \frac{1}{b-a}$$

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{50} & 0 \leq x \leq 50 \\ 0 & x > 50 \end{cases}$$

---

$$P(10 \leq 8 \leq 20) = \int_{10}^{20} f(x)\, dx$$

$$= \int_{10}^{20} \frac{1}{50}\, dx = \frac{1}{50}\left(x \Big|_{10}^{20}\right)$$

$$= \frac{1}{50}(20-10) = \frac{1}{5} = 0.2.$$

Expected value / mean value / expectation:

$\underline{\xi}$: density fct. $f(x)$.

mean value: $\mu = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$.

$\updownarrow$

average.

median value: $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\downarrow$

$\alpha$: $\int_{\alpha}^{\infty} f(x)\,dx = \frac{1}{2}$. $\int_{-\infty}^{\alpha} f(x)\,dx = \frac{1}{2}$

$$f(x) = \begin{cases} 0 & x < a \\ \frac{1}{b-a} & a \le x \le b \\ 0 & x > b \end{cases}$$

---

__mean__ __value__: $\mu = \int_{-\infty}^{+\infty} x \cdot f(x)\,dx = \int_a^b x \cdot f(x)\,dx.$

$$= \int_a^b x \cdot \frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a}\left(\frac{1}{2}x^2 \Big|_a^b\right) = \frac{1}{b-a} \cdot \frac{b^2-a^2}{2}$$

$$= \frac{1}{\cancel{b-a}} \cdot \frac{\cancel{(b-a)}(b+a)}{2} = \frac{b+a}{2}$$

**median value.**

$$\int_{\alpha}^{\infty} f(x)\,dx = \frac{1}{2}$$

$$\int_{\alpha}^{b} f(x)\,dx = \frac{1}{2}$$

$$\int_{\alpha}^{b} \frac{1}{b-a}\,dx = \frac{1}{2}.$$

$$= \frac{1}{b-a}\cdot(b-\alpha) = \frac{1}{2}.$$

$$b-\alpha = \frac{b-a}{2}.$$

**mean V.**

$$\Longleftrightarrow \quad \alpha = \frac{b+a}{2}.$$

$$f(x) = \begin{cases} 0 & x < 0 \\ 192x\left(\frac{1}{2}-x\right)^2 & 0 \le x \le \frac{1}{2} \\ 0 & x > \frac{1}{2} \end{cases}$$

---

<u>mean</u> <u>value</u>: $\mu = \int_{-\infty}^{\infty} x f(x) \cdot dx$

$$= \int_0^{\frac{1}{2}} x \cdot 192x\left(\frac{1}{2}-x\right)^2 dx.$$

$$= 192 \int_0^{1/2} x^2\left(\frac{1}{4}-x+x^2\right) dx.$$

$$= 192 \int_0^{1/2} \left[ \frac{x^2}{4} - x^3 + x^4 \right] dx$$

$$= 192 \left[ \frac{x^3}{12} - \frac{x^4}{4} + \frac{x^5}{5} \right]_0^{1/2}.$$

$$= 192 \left[ \frac{(\frac{1}{2})^3}{12} - \frac{(\frac{1}{2})^4}{4} + \frac{(\frac{1}{2})^5}{5} \right]$$

$$= \frac{1151}{8450} \approx 0.136.$$

$$\frac{1}{2} = \int_{\alpha}^{\infty} f(x)\,dx.$$

$$= \int_{\alpha}^{\frac{1}{2}} f(x)\,dx = \int_{\alpha}^{1/2} 192\, x\left(\tfrac{1}{4} - x + x^2\right)dx$$

$$= 192 \int_{\alpha}^{1/2} \left(\frac{x}{4} - x^2 + x^3\right) dx$$

$$= 192 \left[\frac{x^2}{8} - \frac{x^3}{3} + \frac{x^4}{4}\right]_{\alpha}^{1/2}.$$

$$= 192 \left[\frac{(1/2)^2}{8} - \frac{(1/2)^3}{3} + \frac{(1/2)^4}{4}\right] -$$

$$192\left[\frac{\alpha^2}{8}-\frac{\alpha^3}{3}+\frac{\alpha^4}{4}\right]$$

$$\frac{1}{2}=1-192\left[\frac{\alpha^2}{8}-\frac{\alpha^3}{3}+\frac{\alpha^4}{4}\right]$$

$$\frac{\alpha^2}{8}-\frac{\alpha^3}{3}+\frac{\alpha^4}{4}=\frac{1}{2\times 192}.$$

By cal

$\alpha=\underline{0.193} \Leftarrow$ <u>median value</u>.