# Review

## Ex 1

① $\int x^2 \ln x \, dx$

$$\left( \begin{array}{ll} u = \ln x & v = \frac{1}{3}x^3 \\ du = \frac{1}{x}\,dx & dv = x^2\,dx \end{array} \right)$$

$$= \ln x \cdot \frac{1}{3}x^3 - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3} \cdot \frac{1}{3}x^3 + C$$

② $\int x^2 \sin(x^3-1)\,dx$

$$\left(u = x^3 - 1 \qquad du = 3x^2\,dx\right)$$

$$= \frac{1}{3}\int \sin(x^3-1)\,\underline{3x^2\,dx}$$

$$= \frac{1}{3}\int \sin u\,du = \frac{1}{3}(-\cos u) + C$$

$$= -\frac{1}{3}\cos(x^3-1) + C$$

(3) $\int \sin(\sin x) \cdot \underline{\cos x \, dx}$

$\left( u = \sin x \qquad du = \cos x \, dx \right)$

$= \int \sin u \, du$

$= -\cos u + C$

$= -\cos(\sin x) + C$

(4) $\displaystyle\int \frac{e^{2x}}{(1+e^{2x})^{3/2}}\,dx$

$(u = 1+e^{2x} \qquad du = 2e^{2x}\,dx)$

$$= \frac{1}{2}\int \frac{1}{(1+e^{2x})^{3/2}}\,\underline{2e^{2x}\,dx}$$

$$= \frac{1}{2}\int \frac{1}{u^{3/2}}\,du = \frac{1}{2}\cdot\frac{1}{-1/2}u^{-1/2} + C$$

$$= -(1+e^{2x})^{-1/2} + C$$

⑤ $\int (x^2+2x)e^{2x}\,dx$

$$\begin{pmatrix} u = x^2+2x & v = \frac{1}{2}e^{2x} \\ du = (2x+2)dx & dv = e^{2x}dx \end{pmatrix}$$

$= (x^2+2x)\cdot\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}\cdot(2x+2)dx$

$= \frac{1}{2}(x^2+2x)e^{2x} - \int (x+1)e^{2x}dx$

$$\begin{pmatrix} u = x+1 & v = \frac{1}{2}e^{2x} \\ du = dx & dv = e^{2x}dx \end{pmatrix}$$

$$= \frac{1}{2}(x^2+2x)e^{2x} - \left[(x+1)\cdot\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,dx\right]$$

$$= \frac{1}{2}(x^2+2x)e^{2x} - \frac{1}{2}(x+1)e^{2x} + \frac{1}{2}\int e^{2x}\,dx$$

$$= \frac{1}{2}(x^2+2x)e^{2x} - \frac{1}{2}(x+1)e^{2x} + \frac{1}{2}\cdot\frac{1}{2}e^{2x} + C$$

⑥ $\int_0^2 x e^{-x^2}\,dx$

method 1:

$\int x e^{-x^2}\,dx$

$(u = -x^2 \quad du = -2x\,dx)$

$= \frac{1}{-2}\int e^{-x^2}\underline{(-2)x\,dx}$

$= -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C$

$= -\frac{1}{2}e^{-x^2} + C$

$$\int_0^2 x e^{-x^2} dx = -\frac{1}{2} e^{-x^2} \Big|_0^2$$

$$= -\frac{1}{2}\left(e^{-4} - 1\right)$$

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method 2:

$x: 0 \to 2$

$u: 0 \to -4$

$$\int_0^2 x e^{-x^2} dx$$

$$\left(u = -x^2 \quad du = -2x\,dx\right)$$

$$= -\frac{1}{2}\int_0^{-4} e^u du = -\frac{1}{2} e^u \Big|_0^{-4}$$

$$= -\frac{1}{2}\left(e^{-4} - 1\right)$$

7 $\int_0^{\pi} x \cos 2x \, dx$

$$\left( \begin{array}{ll} u = x & v = \frac{1}{2}\sin 2x \\ du = dx & dv = \cos 2x \, dx \end{array} \right)$$

$$= x \cdot \frac{1}{2}\sin 2x \Big|_0^{\pi} - \int_0^{\pi} \frac{1}{2}\sin 2x \, dx$$

$$= \pi \cdot \frac{1}{2}\sin 2\pi - 0 - \frac{1}{2} \cdot \frac{1}{2} \cdot \left(-\cos(2x)\right) \Big|_0^{\pi}$$

$$= \frac{1}{4}\left(\cos(2\pi) - \cos 0\right) = 0$$

⑧ $\int x(x^3-1)^2\,dx$

$= \int x\cdot(x^6-2x^3+1)\,dx$

$= \int (x^7-2x^4+x)\,dx$

$= \frac{1}{8}x^8 - 2\cdot\frac{1}{5}x^5 + \frac{1}{2}x^2 + C.$

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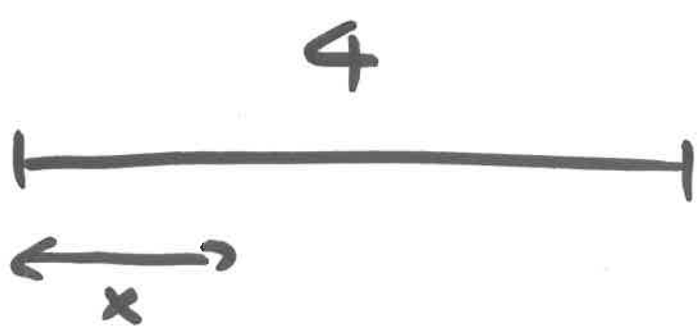
· $\int x^2(x^3-1)^{10}\,dx$

$u = x^3-1$, $du = 3x^2\,dx$

<u>Ex 2</u> A rod has length 4 meters. Let the density given by

$f(x) = 2 + \sin(\pi x)$ (in kg/meter)

where $x$ is the distance to one end. Compute average density.



$$\bar{f} = \frac{1}{4-0}\int_0^4 f(x)\,dx$$

$$= \frac{1}{4}\int_0^4 \left(2 + \sin(\pi x)\right) dx$$

$$= \frac{1}{4}\left(2x - \frac{1}{\pi}\cos(\pi x)\right)\Big|_0^4$$

$$= \frac{1}{4}\left[\left(2\cdot 4 - \frac{1}{\pi}\cos(4\pi)\right) - \left(0 - \frac{1}{\pi}\cos 0\right)\right]$$

$$= \frac{1}{4}\left[8 - \frac{1}{\pi} + \frac{1}{\pi}\right] = 2$$

Ex 3

(1) $\int x \ln(x^2+1)\,dx$

$$\left( y = x^2+1 \qquad dy = 2x\,dx \right)$$

$$= \frac{1}{2}\int \ln(x^2+1)\cdot 2x\,dx$$

$$= \frac{1}{2}\int \ln y\,dy$$

$$\begin{pmatrix} u = \ln y & v = y \\ du = \frac{1}{y}dy & dv = dy \end{pmatrix}$$

$$= \frac{1}{2}\left( (\ln y)\cdot y - \int y \cdot \frac{1}{y}\, dy \right)$$

$$= \frac{1}{2}\left( y \ln y - y \right) + C$$

$$= \frac{1}{2}\left( (x^2+1)\ln (x^2+1) - (x^2+1) \right) + C$$

② $\int e^x \sin x \, dx$

$$\left( \begin{array}{ll} u = e^x & v = -\cos x \\ du = e^x dx & dv = \sin x \, dx \end{array} \right)$$

$$= -e^x \cos x - \int (-\cos x) \cdot e^x dx$$

$$= -e^x \cos x + \int e^x \cos x \, dx$$

$$\left( \begin{array}{ll} u = e^x & v = \sin x \\ du = e^x dx & dv = \cos x \, dx \end{array} \right)$$

$$= -e^x\cos x + e^x\sin x - \int \sin x \cdot e^x\,dx$$

$$= -e^x\cos x + e^x\sin x - \int e^x\sin x\,dx$$

$$A = \int e^x\sin x\,dx$$

$$A = -e^x\cos x + e^x\sin x - A$$

$$2A = -e^x\cos x + e^x\sin x$$

$$A = \frac{1}{2}(-e^x\cos x + e^x\sin x)$$

$$\int e^x\sin x\,dx = \frac{1}{2}(-e^x\cos x + e^x\sin x) + C$$