Review

Ex 1 Sketch graph

$$f(x) = x e^{-\frac{x^2}{2}}$$

domain ✓

$$f'(x) = 1 \cdot e^{-\frac{x^2}{2}} + x \cdot e^{-\frac{x^2}{2}} \cdot (-x)$$

$$= e^{-\frac{x^2}{2}} (1 - x^2)$$

$$f'(x) = 0 \Rightarrow \quad 1 - x^2 = 0$$

$$x^2 = 1 \qquad x = 1, -1$$

$$\begin{array}{ccccc} - & & + & & - \\ \hline & -1 & & 1 & \rightarrow \end{array}$$

$$f''(x) = e^{-\frac{x^2}{2}} \cdot (-x) \cdot (1 - x^2)$$

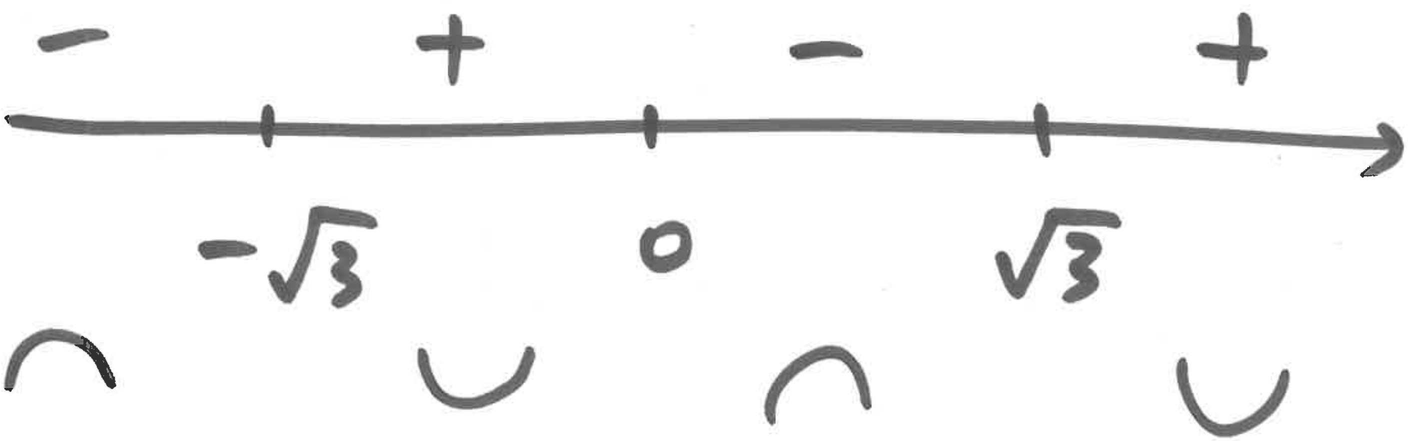
$$+ e^{-\frac{x^2}{2}} \cdot (-2x)$$

$$= e^{-\frac{x^2}{2}} \left(-x + x^3 - 2x \right)$$

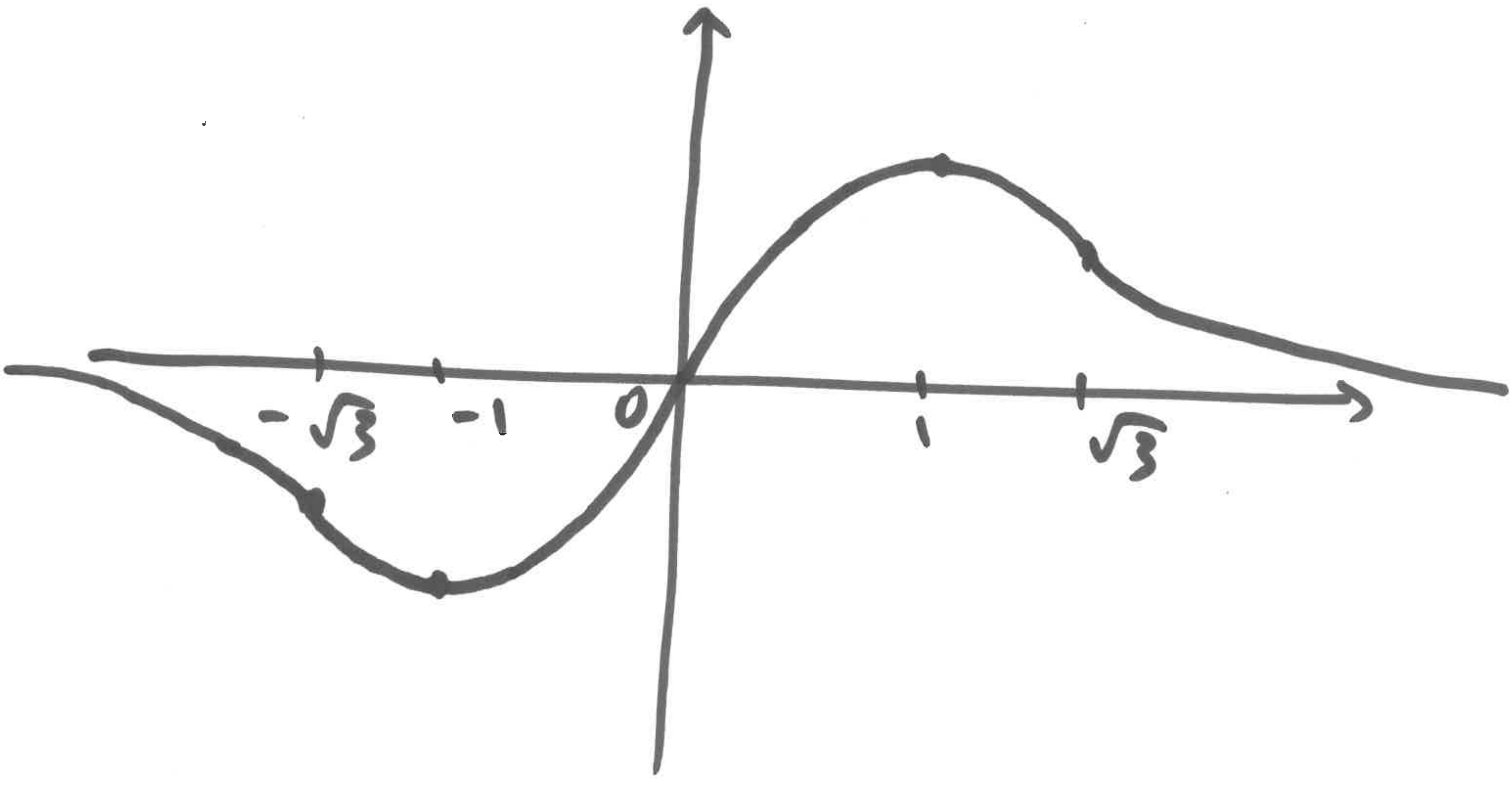
$f''(x) = 0 \Rightarrow x^3 - 3x = 0$

$x(x^2 - 3) = 0$

$x = 0, \sqrt{3}, -\sqrt{3}$



$$\lim_{x \to \infty} x e^{-x^2} = 0, \lim_{x \to -\infty} x e^{-x^2} = 0$$



Ex 2 Find global max/min of $f(x) = x - x^3$ on $[-2, 0]$

$$f'(x) = 1 - 3x^2 = 0$$

$$3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$x = \pm \frac{1}{\sqrt{3}} \qquad \underbrace{x = -\frac{1}{\sqrt{3}}}$$

$$f(-2) = -2 - (-2)^3 = 6 \quad \text{max}$$

$$f(0) = 0$$

$$f\left(-\frac{1}{\sqrt{3}}\right) = -\frac{1}{\sqrt{3}} - \left(-\frac{1}{\sqrt{3}}\right)^3$$

$$= -\frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}}$$

$$= -\frac{2}{3} \cdot \frac{1}{\sqrt{3}} \quad \text{min}$$

Ex 3 Sketch graph

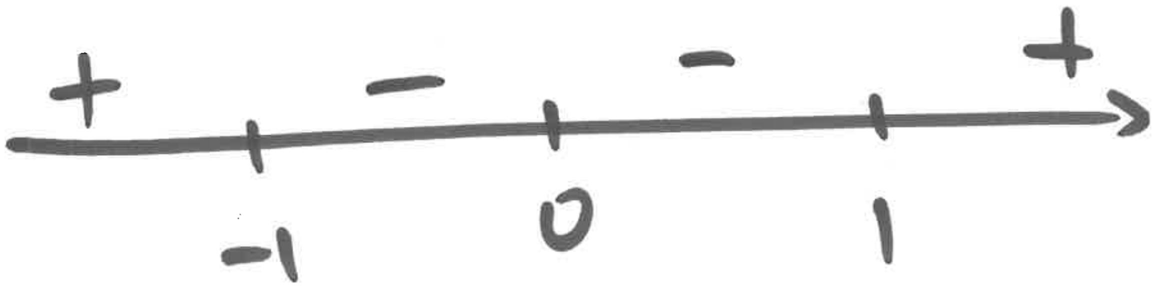
$$f(x) = \frac{x^2 - x + 1}{x} = x - 1 + \frac{1}{x}$$

domain : $x \neq 0$

$$f'(x) = 1 - \frac{1}{x^2}$$

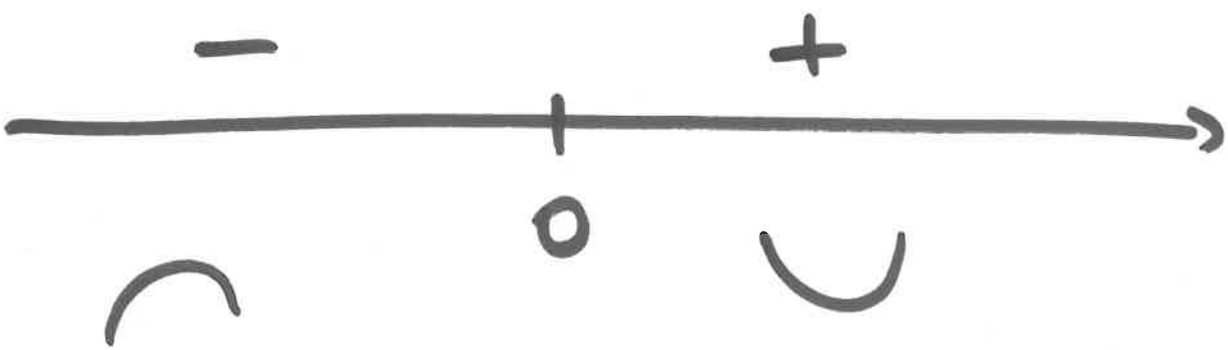
$$f'(x) = 0 \Rightarrow 1 = \frac{1}{x^2} \qquad x^2 = 1$$

$$x = \pm 1$$



$$f''(x) = -(-2)\cdot\frac{1}{x^3} = \frac{2}{x^3}$$

$$f''(x) \neq 0$$

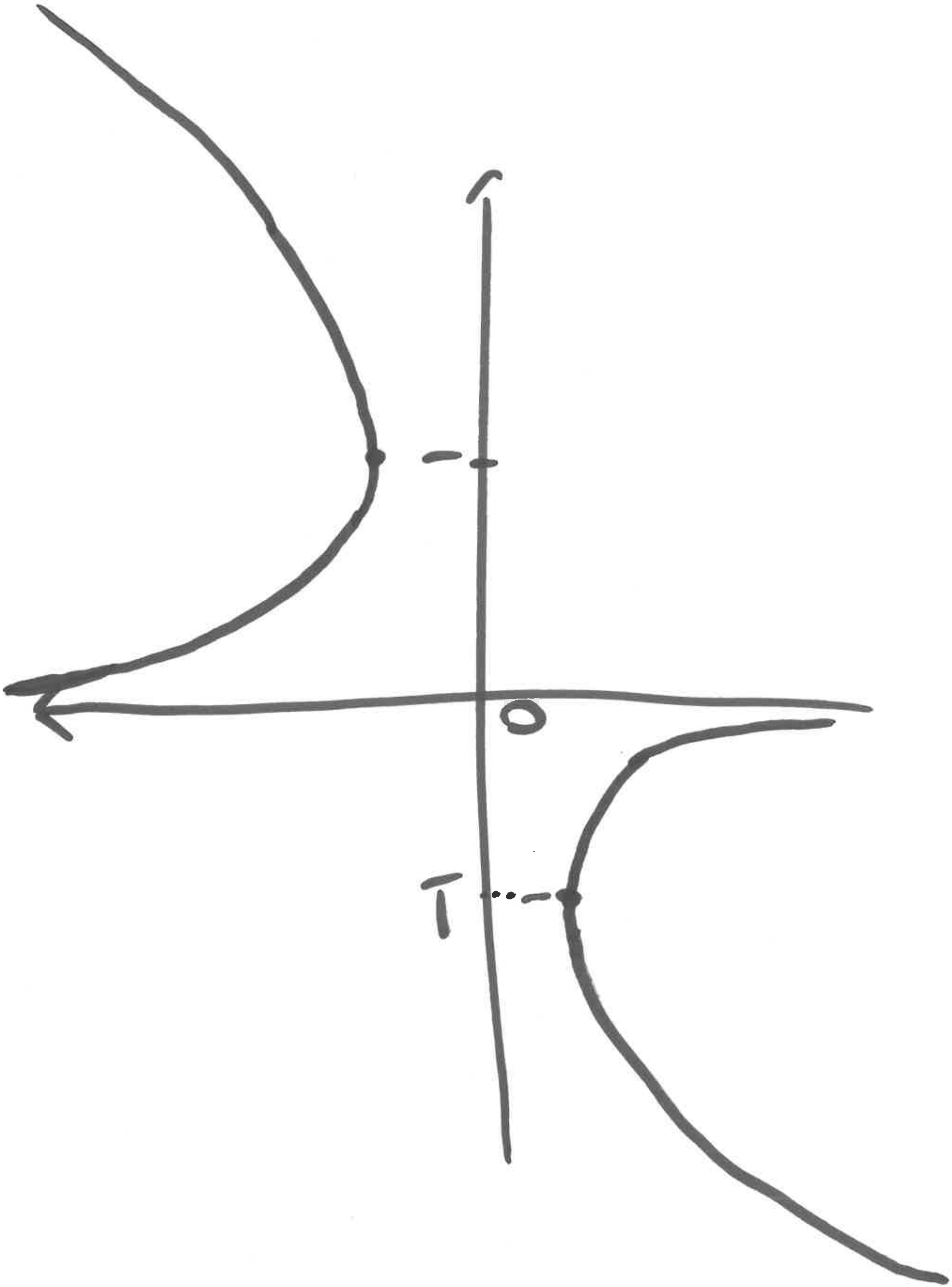


$$\lim_{x \to \infty} \left(x - 1 + \frac{1}{x} \right) = \infty$$

$$\lim_{x \to -\infty} \left(x - 1 + \frac{1}{x} \right) = -\infty$$

$$\lim_{x \to 0+} \left(x - 1 + \frac{1}{x} \right) = \infty$$

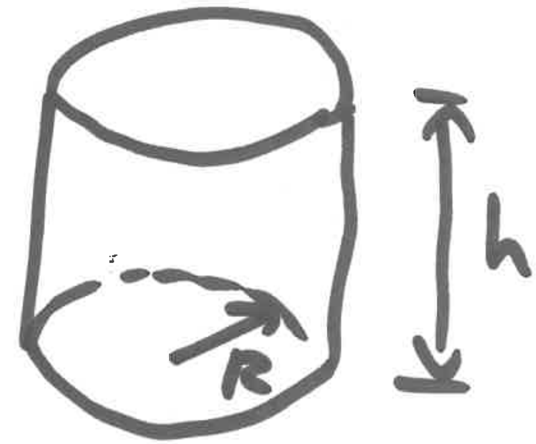
$$\lim_{x \to 0-} \left(x - 1 + \frac{1}{x} \right) = -\infty$$



Ex 4 Make a cylindrical can, without top, volume $= 10$. What is the least amount of material to use?

(volume $= \pi R^2 h$, side area $= 2\pi R \cdot h$)

$$A = 2\pi R h + \pi R^2$$



Know: $V = \pi R^2 h = 10$

$$h = \frac{10}{\pi R^2}$$

$$A(R) = 2\pi R \cdot \frac{10}{\pi R^2} + \pi R^2$$

$$= \frac{20}{R} + \pi R^2$$

want to minimize

Domain: $(0, \infty)$

$$A'(R) = -\frac{20}{R^2} + 2\pi R = 0$$

$$2\pi R = \frac{20}{R^2}$$

$$R^3 = \frac{20}{2\pi} = \frac{10}{\pi}$$

$$R = \sqrt[3]{\frac{10}{\pi}}$$

$$A\left(\sqrt[3]{\frac{10}{\pi}}\right) = \frac{20}{\sqrt[3]{\frac{10}{\pi}}} + \pi\left(\sqrt[3]{\frac{10}{\pi}}\right)^2$$

To justify $A\left(\sqrt[3]{\frac{10}{\pi}}\right)$ is min,

$$\lim_{R \to \infty} \left(\frac{20}{R} + \pi R^2\right) = \infty$$

$$\lim_{R \to 0^+} \left(\frac{20}{R} + \pi R^2\right) = \infty$$

Ex 5 Approximate $\int_{-1}^{1} \sqrt{1+x^2}\, dx$ by using 5 trapezoids.

$$\Delta x = \frac{1-(-1)}{5} = \frac{2}{5} = 0.4$$

i	0	1	2	3	4	5
x_i	-1	-0.6	-0.2	0.2	0.6	1
$f(x_i)$	$\sqrt{2}$	$\sqrt{1.36}$	$\sqrt{1.04}$	$\sqrt{1.04}$	$\sqrt{1.36}$	$\sqrt{2}$

$$\int_{-1}^{1} \sqrt{1+x^2}\,dx \approx \frac{1-(-1)}{2\cdot 5}\Big(\sqrt{2}$$

$$+ 2\left(\sqrt{1.36} + \sqrt{1.04} + \sqrt{1.04} + \sqrt{1.36}\right)$$

$$+ \sqrt{2}\Big)$$

__Ex 6__ Let $f(x) = \int_{\boxed{e^{-x}}}^{1} \sqrt{1+y^2}\, dy$.

Compute $f'(x)$.

Recall $\frac{d}{dx} \int_a^x g(y)\, dy = g(x)$

$\frac{d}{dx} \int_x^b g(y)\, dy = -g(x)$

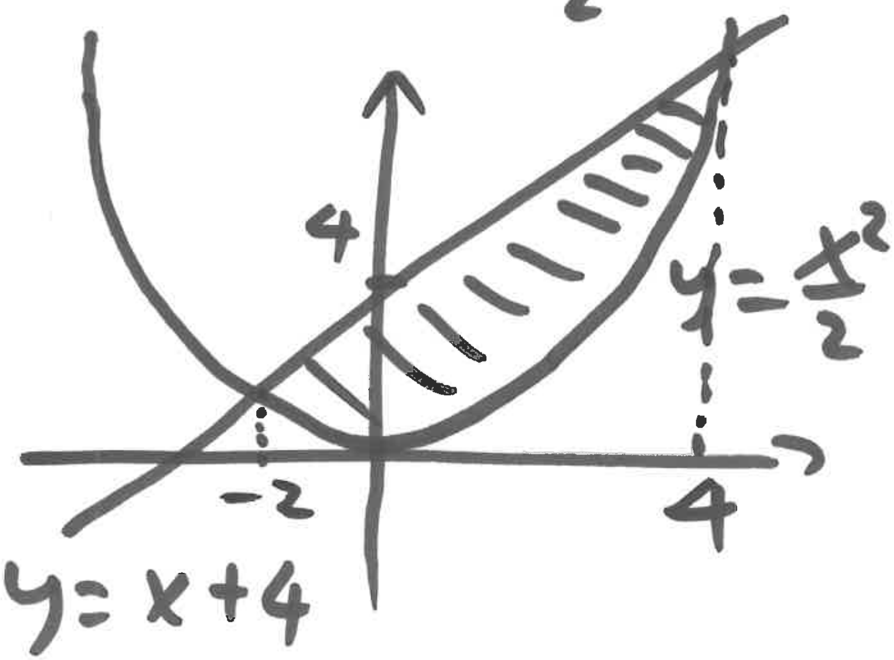
$$f'(x) = \frac{df}{d\square} \cdot \frac{d\square}{dx}$$

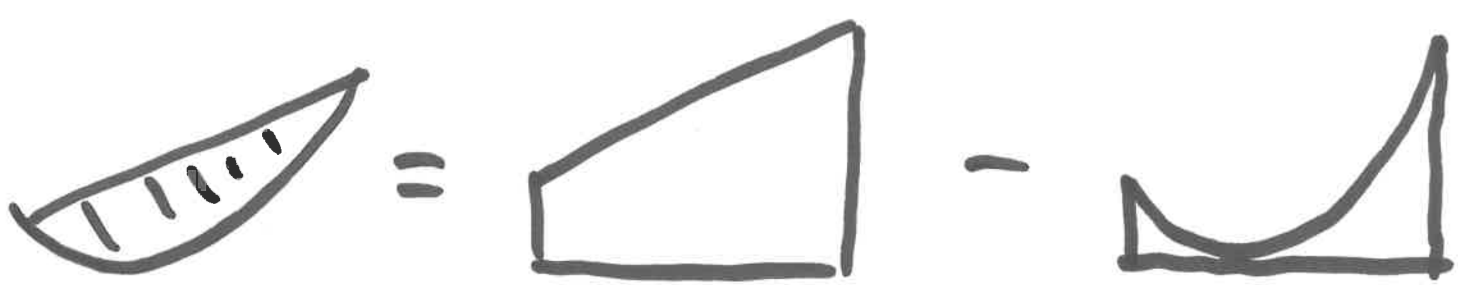
$$= -\sqrt{1+\square^2} \cdot (-e^{-x})$$

$$= e^{-x} \cdot \sqrt{1+e^{-2x}}$$

__Ex 7__ Compute area bounded by

$$y = \frac{x^2}{2} \quad \text{and} \quad y = x + 4$$





intersection pts:

$$\frac{x^2}{2} = x + 4$$

$$x^2 = 2x + 8$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4, -2$$

$$A = \int_{-2}^{4} \left(x + 4 - \frac{x^2}{2} \right) dx$$

$$= \left(\frac{1}{2}x^2 + 4x - \frac{1}{2} \cdot \frac{1}{3}x^3 \right) \Big|_{-2}^{4}$$

$$= \left(\frac{1}{2}4^2 + 4 \cdot 4 - \frac{1}{6}4^3 \right)$$

$$- \left(\frac{1}{2}(-2) + 4 \cdot (-2) - \frac{1}{6}(-2)^3 \right)$$

$$= 18$$