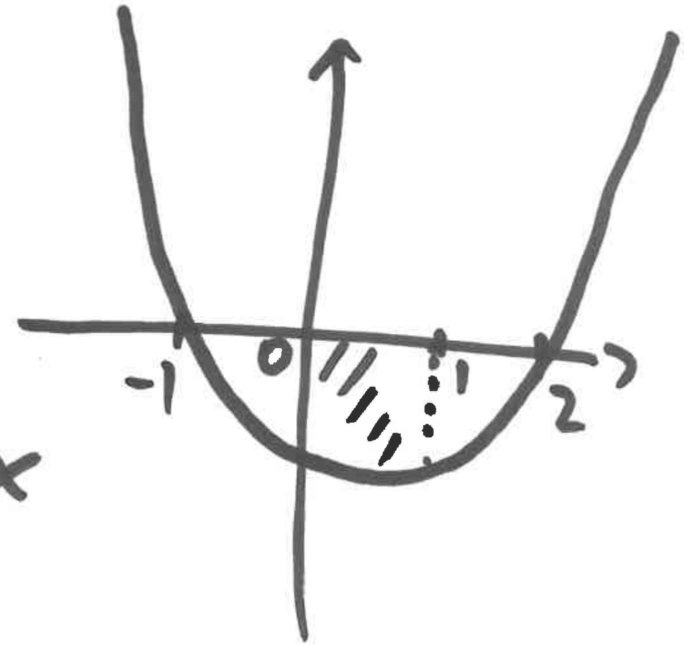
22.1 ~ 22.4 (Continued)



Ex 1 Compute

① $\int_0^1 (x+1)(x-2)\,dx$

$$= \int_0^1 (x^2 - x - 2)\,dx$$

$$= \left(\frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x\right)\Big|_0^1$$

$$= \left(\frac{1}{3} - \frac{1}{2} - 2\right) - 0 = -\frac{13}{6}$$

(2) $\int 2^x \, dx$

$$= \int e^{\ln(2^x)} \, dx$$

$$= \int e^{x \cdot \ln 2} \, dx$$

$$= \frac{1}{\ln 2} e^{x \cdot \ln 2} + C$$

$$= \frac{1}{\ln 2} 2^x + C$$

③ $$\int \frac{x+1}{\sqrt{x}}\,dx$$

$$= \int \left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} \right) dx$$

$$= \int \left(x^{1/2} + x^{-1/2} \right) dx$$

$$= \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$$

- $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

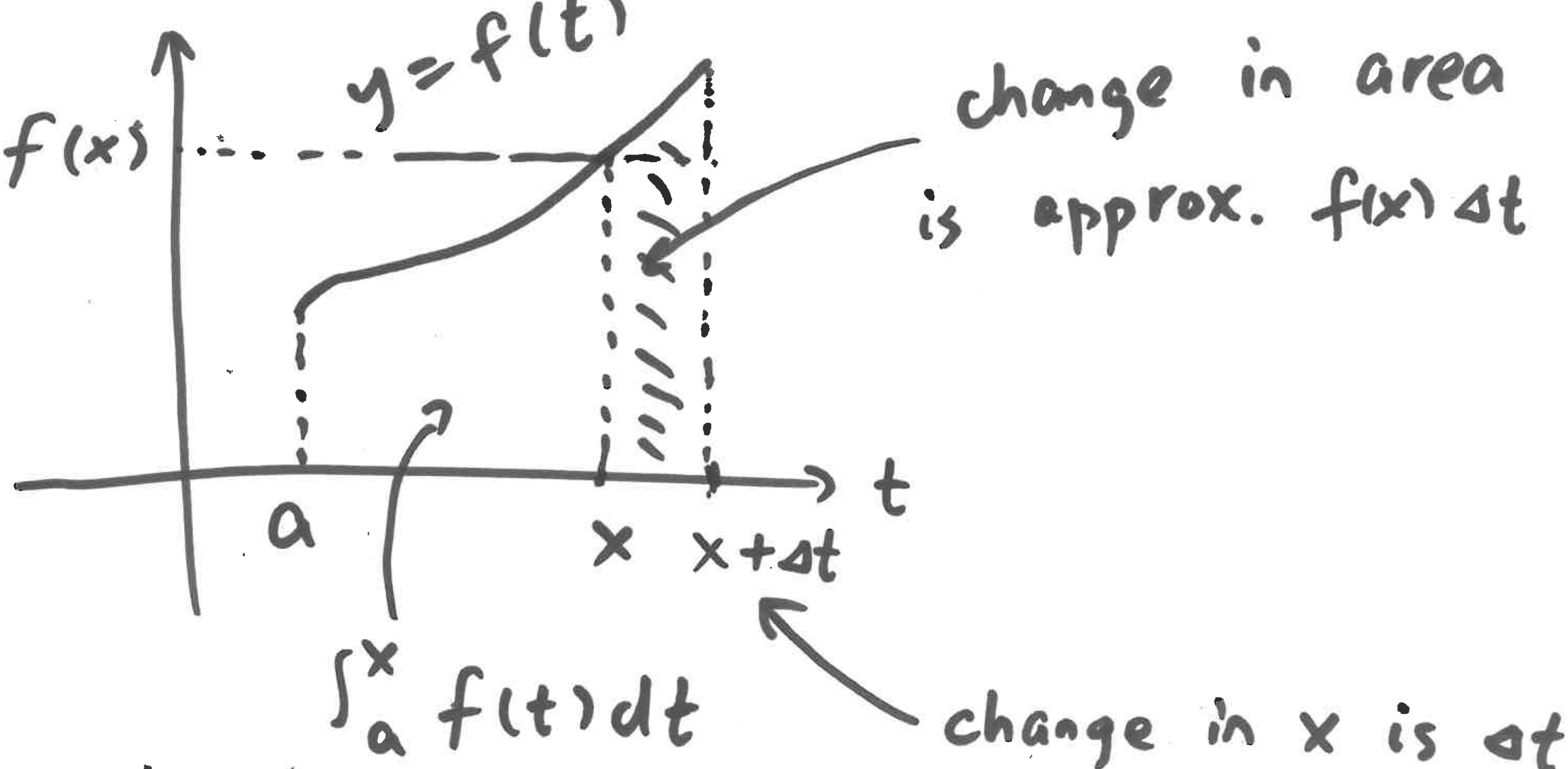
$\frac{d}{dx}\int_x^b f(t)\,dt = -f(x)$

Let $F(t)$ be an anti-der. of $f(t)$

$$\int_a^x f(t)\,dt = F(x) - F(a)$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = \frac{d}{dx}\left(F(x) - F(a)\right)$$

$$= F'(x) = f(x)$$



y = f(t)
f(x)
change in area is approx. f(x) Δt
t
a
x
x + Δt
∫_a^x f(t) dt
change in x is Δt

Ex 2 Compute

① $$\frac{d}{dx}\int_0^x \sin t \, dt = \sin x$$

check: $$\int_0^x \sin t \, dt = -\cos t \Big|_0^x$$

$$= -\cos x - (-\cos 0)$$

$$= -\cos x + 1$$

$$\frac{d}{dx}(-\cos x + 1) = \sin x$$

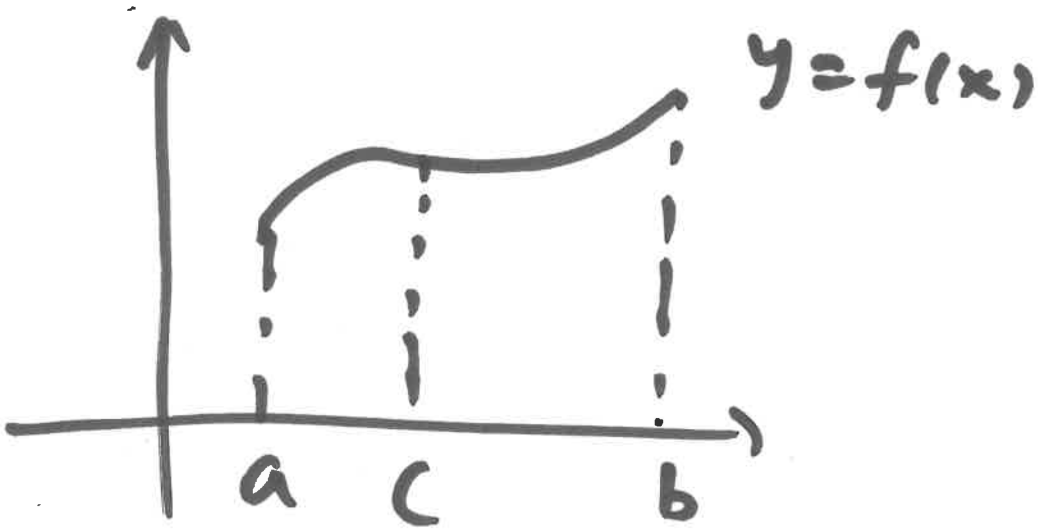
(2) $\dfrac{d}{dx}\displaystyle\int_1^{\boxed{e^x}} \frac{1}{1+t^2}\,dt$

$$= \frac{d}{d\square}\int_1^{\boxed{e^x}} \frac{1}{1+t^2}\,dt \cdot \frac{d\,e^x}{dx}$$

$$= \frac{1}{1+(e^x)^2}\cdot e^x$$

$$= \frac{e^x}{1+e^{2x}}$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



③ $$\frac{d}{dx}\int_{x^2}^{x} 2t\,dt$$

$$= \frac{d}{dx}\left(\int_{\boxed{x^2}}^{0} 2t\,dt + \int_{0}^{x} 2t\,dt\right)$$

$$= \underbrace{-2x^2}_{\text{"}-f(\square)\text{"}} \cdot \underbrace{2x}_{\frac{dx^2}{dx}} + 2x$$

$$= -4x^3 + 2x$$

- Integral with absolute values

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

Ex 3 Compute

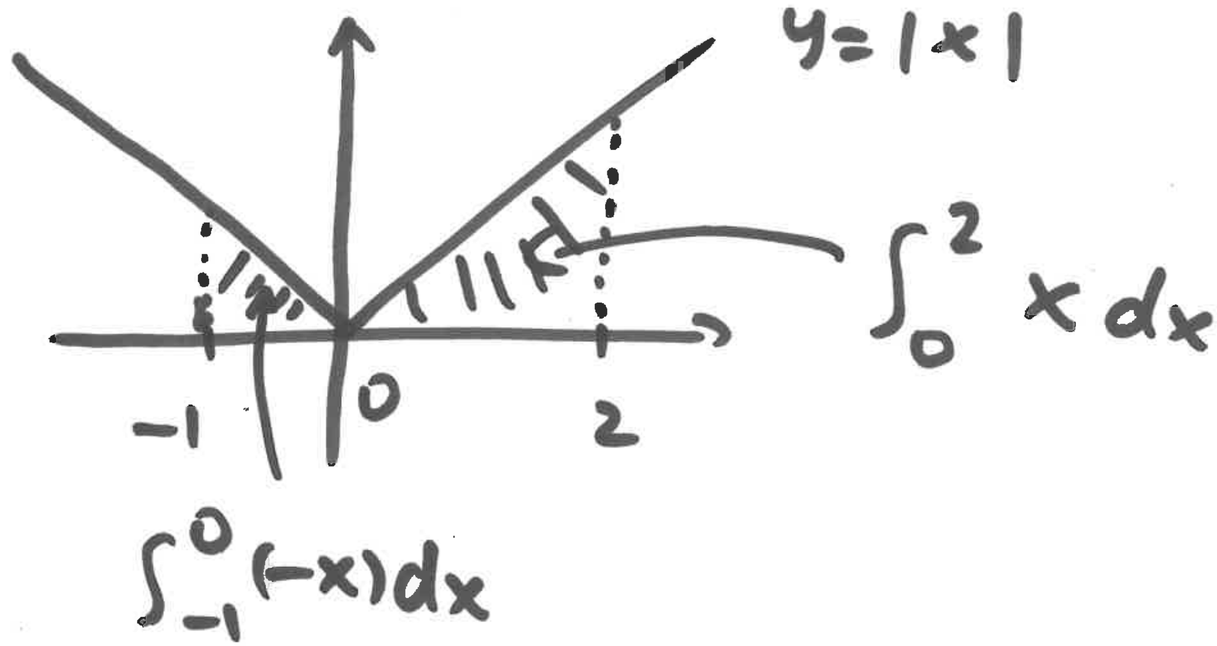
① $\displaystyle\int_{-1}^{2} |x|\, dx$

$$= \int_{-1}^{0} (-x)\, dx + \int_{0}^{2} x\, dx$$

$$= -\frac{1}{2}x^2 \Big|_{-1}^{0} + \frac{1}{2}x^2 \Big|_{0}^{2}$$

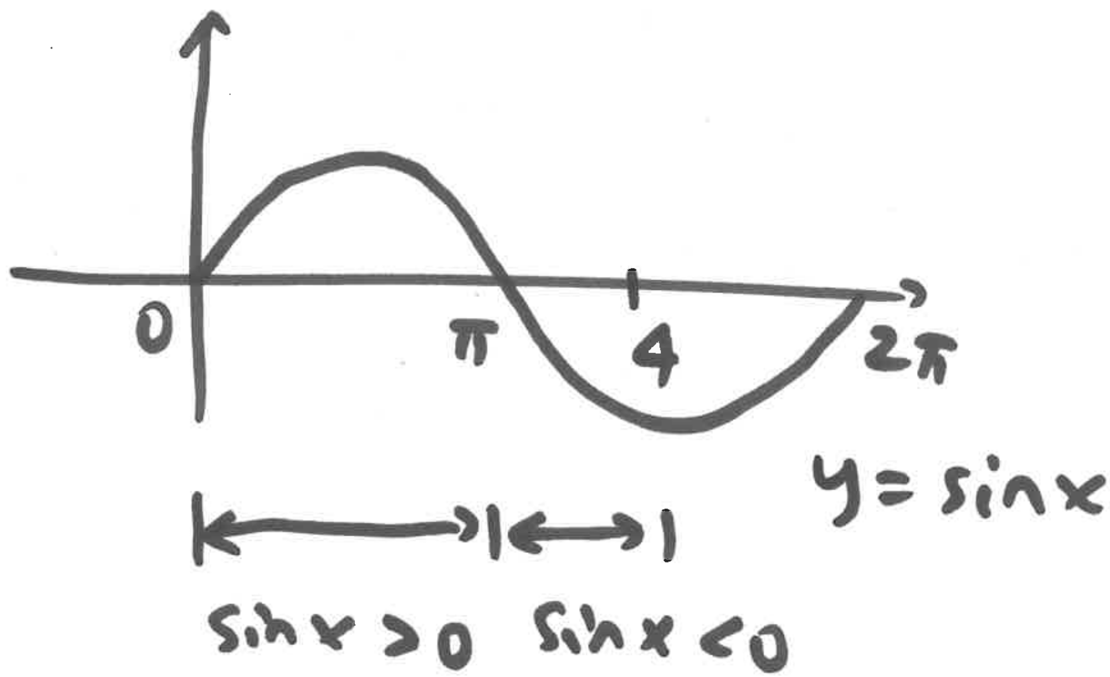
$$= 0 - \left(-\frac{1}{2}\cdot 1\right) + \frac{1}{2}2^2 - 0$$

$$= \frac{5}{2}$$



② $\displaystyle\int_0^4 |\sin x|\, dx$

$$|\sin x| = \begin{cases} \sin x & \sin x \geq 0 \\ -\sin x & \sin x < 0 \end{cases}$$



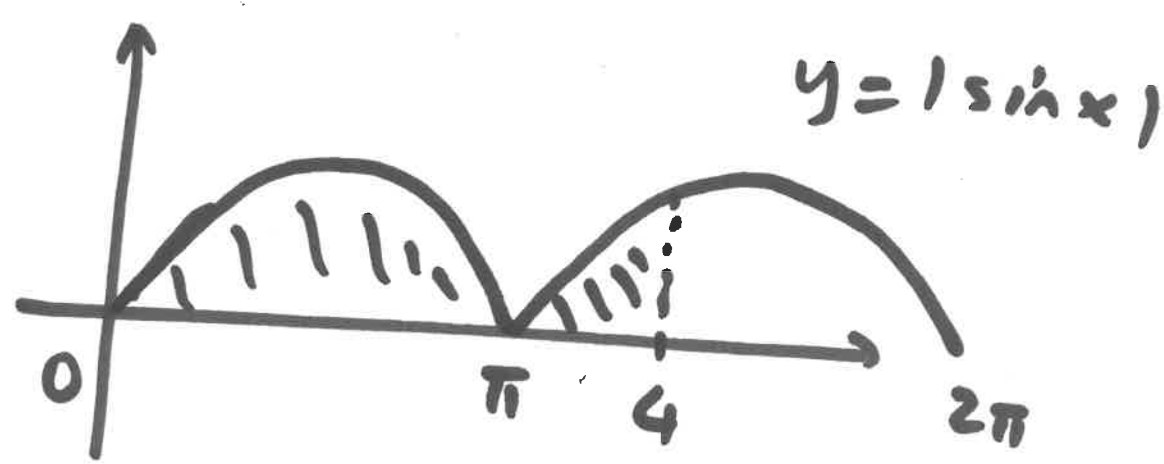
$$\int_0^4 |\sin x|\, dx$$

$$= \int_0^{\pi} \sin x\, dx + \int_{\pi}^{4} (-\sin x)\, dx$$

$$= -\cos x \Big|_0^{\pi} + \cos x \Big|_{\pi}^{4}$$

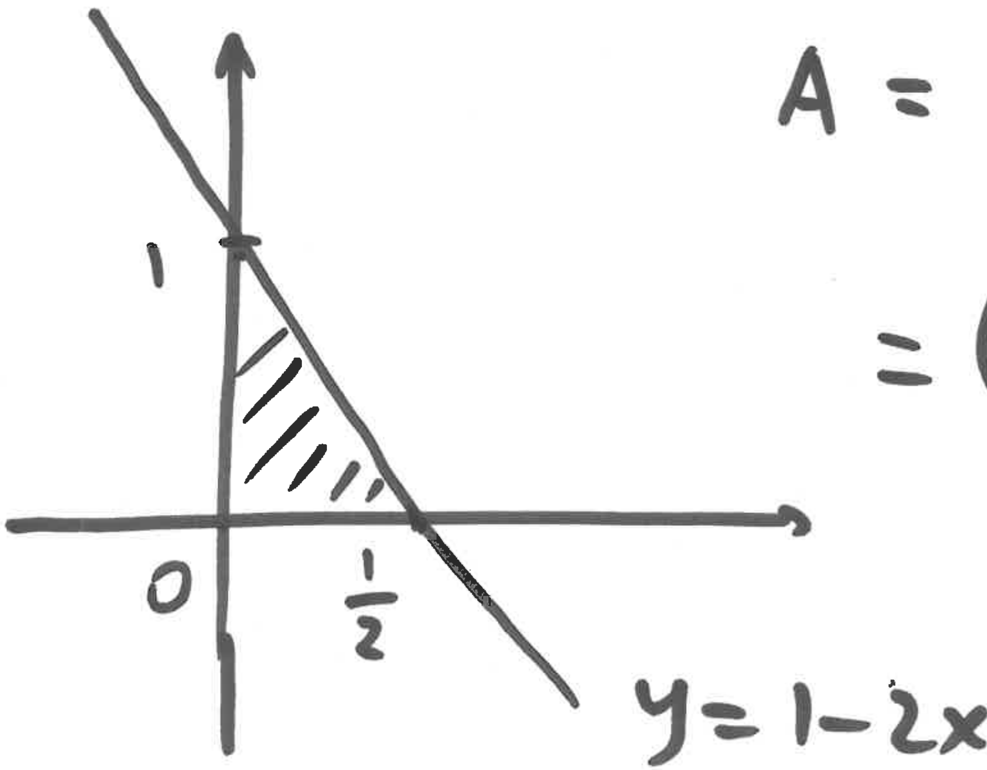
$$= -\cos \pi - (-\cos 0) + \cos 4 - \cos \pi$$

$$= 3 + \cos 4$$



Ex 4 Compute area bounded by:

(1) x-axis, y-axis, $y = 1 - 2x$



$$A = \int_0^{\frac{1}{2}} (1-2x)\,dx$$

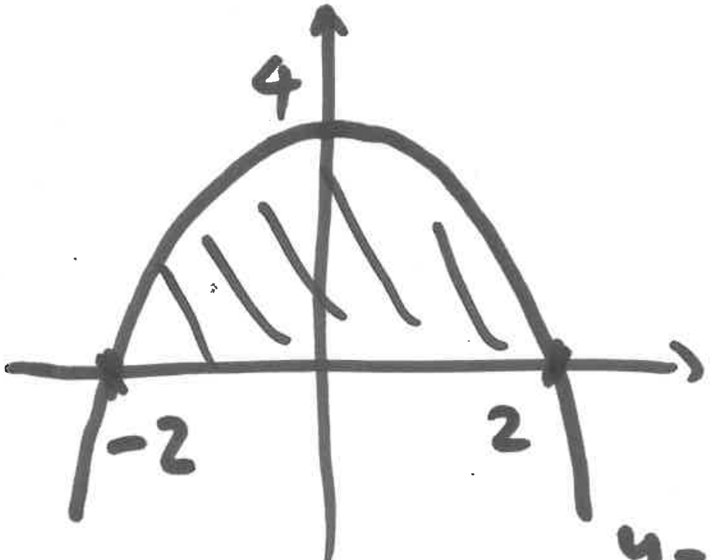
$$= \left(x - 2\cdot\frac{1}{2}x^2\right)\Big|_0^{\frac{1}{2}}$$

$$= \left(\frac{1}{2} - \frac{1}{4}\right) - 0$$

$$= \frac{1}{4}$$

$$A = \frac{1}{2}\cdot\frac{1}{2}\cdot 1 = \frac{1}{4}$$

(2) x-axis, $y = 4 - x^2$



need intersection pts of $y=0$, $y=4-x^2$

$$4 - x^2 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$A = \int_{-2}^{2} (4 - x^2)\, dx$$

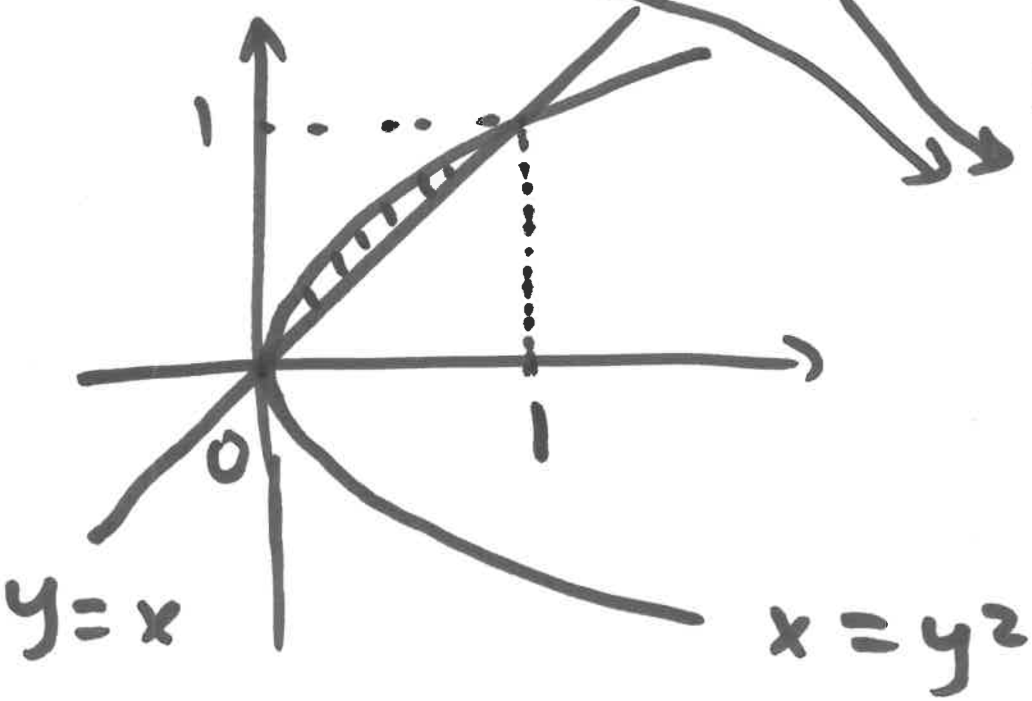
$$= \left(4x - \frac{1}{3}x^3\right)\Big|_{-2}^{2}$$

$$= \left(4 \cdot 2 - \frac{1}{3} 2^3\right) - \left(4(-2) - \frac{1}{3}(-2)^3\right)$$

$$= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)$$

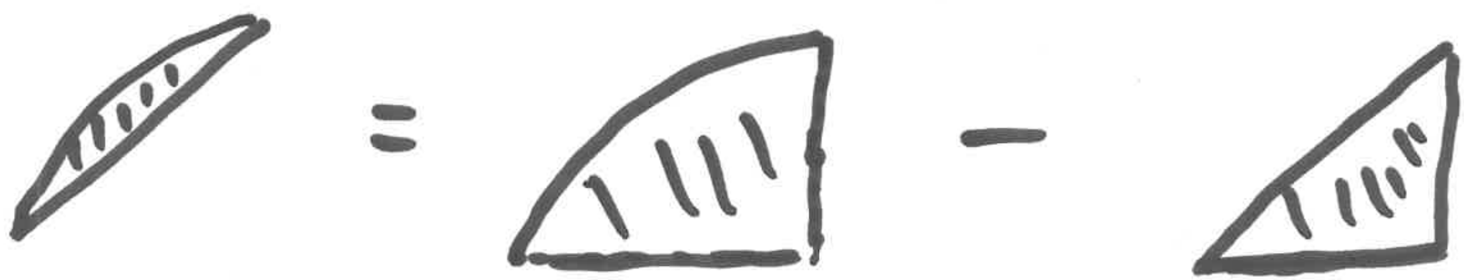
$$= \frac{32}{3}$$

③ $x = y^2, \quad y = x$



intersection:

$$x = x^2$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0, 1$$



$$A = \int_0^1 \sqrt{x}\,dx - \int_0^1 x\,dx$$

$$\left(x = y^2 \Rightarrow y = \sqrt{x} \right)$$

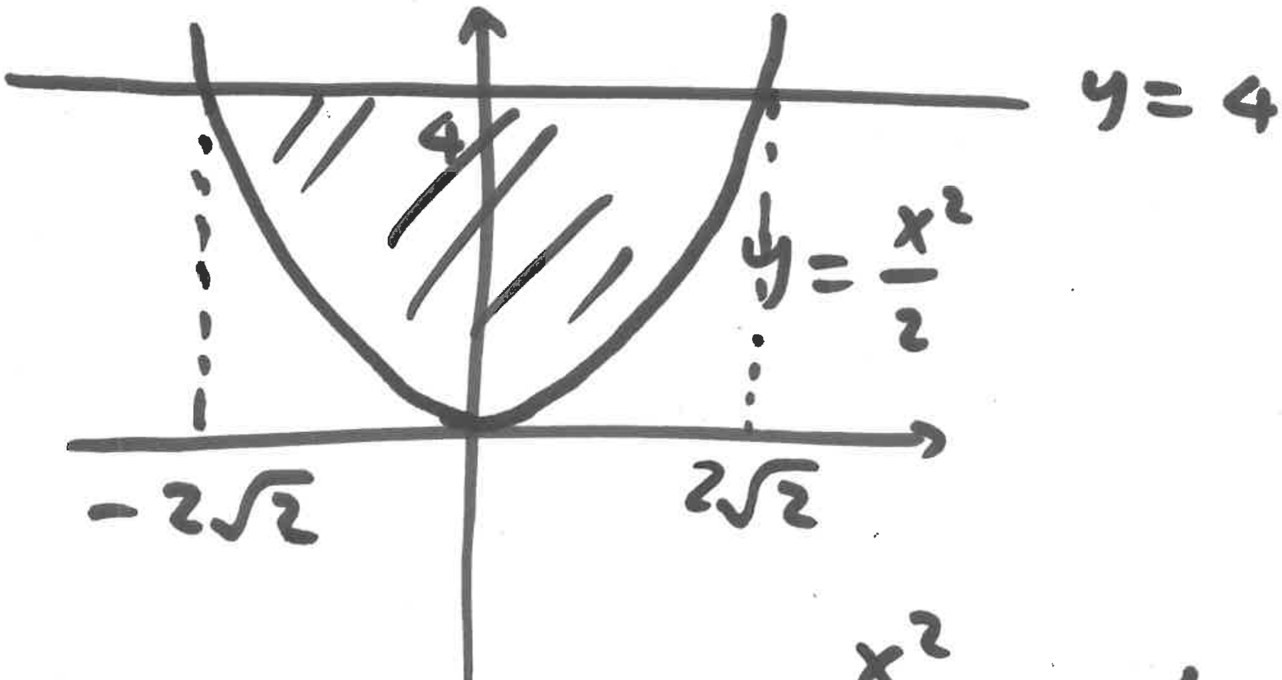
positive, because the part is above x-axis.

$$= \int_0^1 (\sqrt{x} - x)\,dx$$

$$= \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2\right)\Big|_0^1 = \left(\frac{2}{3} - \frac{1}{2}\right) - 0$$

$$= \frac{1}{6}$$

④ $y = \frac{x^2}{2}$, $y = 4$



intersection: $\frac{x^2}{2} = 4$

$x^2 = 8$ $\quad x = \pm 2\sqrt{2}$

$$A = \int_{-2\sqrt{2}}^{2\sqrt{2}} 4\,dx - \int_{-2\sqrt{2}}^{2\sqrt{2}} \frac{x^2}{2}\,dx$$

$$= \int_{-2\sqrt{2}}^{2\sqrt{2}} \left(4 - \frac{x^2}{2}\right) dx$$

$$= \left(4x - \frac{1}{2}\cdot\frac{1}{3}x^3\right)\Big|_{-2\sqrt{2}}^{2\sqrt{2}}$$

$$= \left(4\cdot 2\sqrt{2} - \frac{1}{6}(2\sqrt{2})^3\right)$$

$$- \left(4\cdot(-2\sqrt{2}) - \frac{1}{6}(-2\sqrt{2})^3\right)$$

$$= \frac{32}{3}\sqrt{2}$$