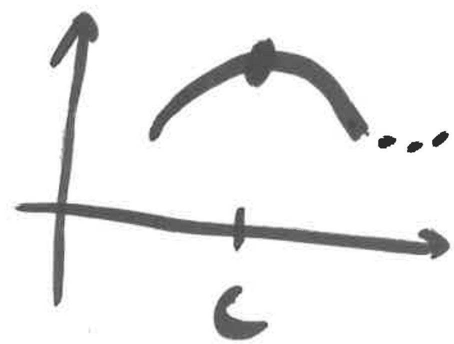
20.1, 20.2 Find max/min

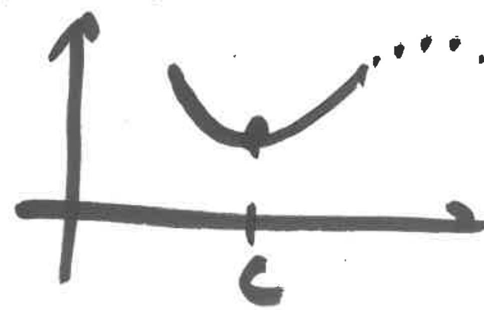
Def $f(x)$ has <u>local maximum</u> at $x = c$ if $f(c) \geq f(x)$ for x near c



.... <u>local minimum</u>

.... $f(c) \leq f(x)$ ...

....



Def $f(x)$ has global maximum at $x=c$ if $f(c) \geq f(x)$ for all x in the domain.

. . . ——— global minimum

. . . . $f(c) \leq f(x)$ . . .

. . . .

Def $f(x)$ is <u>increasing</u> in (a,b)

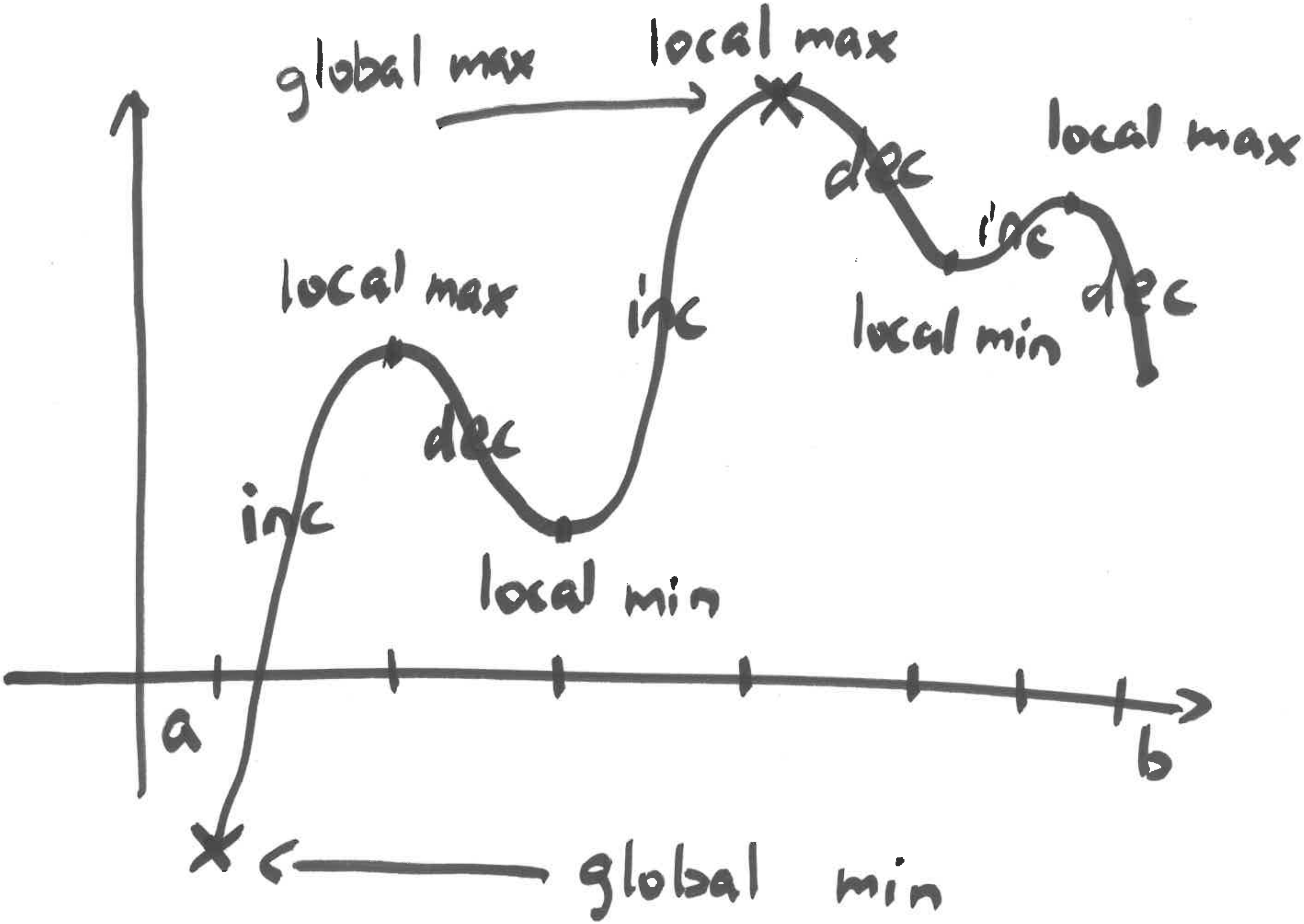
if $f(x) \le f(y)$ whenever

$a \le x \le y \le b$

- - - - - - - - <u>decreasing</u> - - - - - - -

- - - $f(x) \ge f(y)$ - - - - - - -

- - - - - - - -



global max
local max
local max
local max
dec
inc
inc
dec
dec
inc
local min
local min
a
b
global min

First derivative test

$f' > 0$ .... f inc

$f' < 0$ .... f dec

$f' = 0$ .... local max/min

(critical points)

Ex 1 Find inc./dec. intervals, local max/min, sketch graph.

① $f(x) = x^3 - 3x$

$f'(x) = 3x^2 - 3$

$f'(x) = 0 \Rightarrow 3x^2 - 3 = 0$

$x^2 = 1$

$x = 1, \quad x = -1$

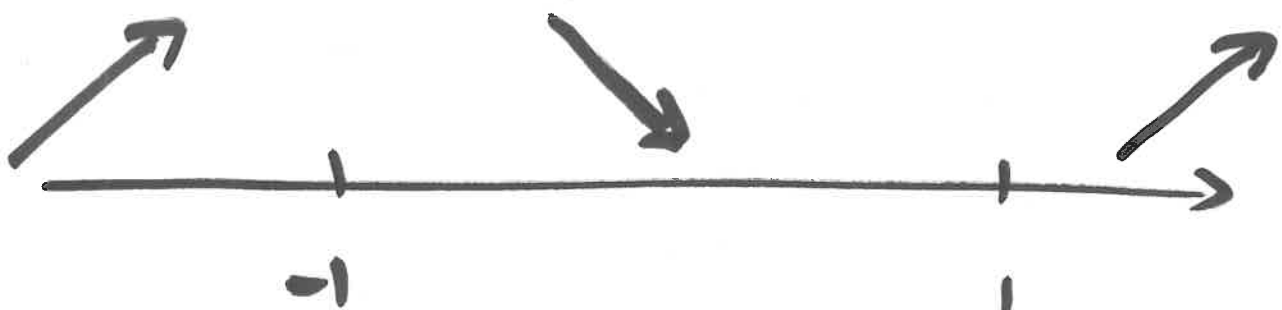
"sign chart"

$f'(-2)=9$

$f'(0)=-3$

$f'(2)=9$

For $f(x)$:



local max

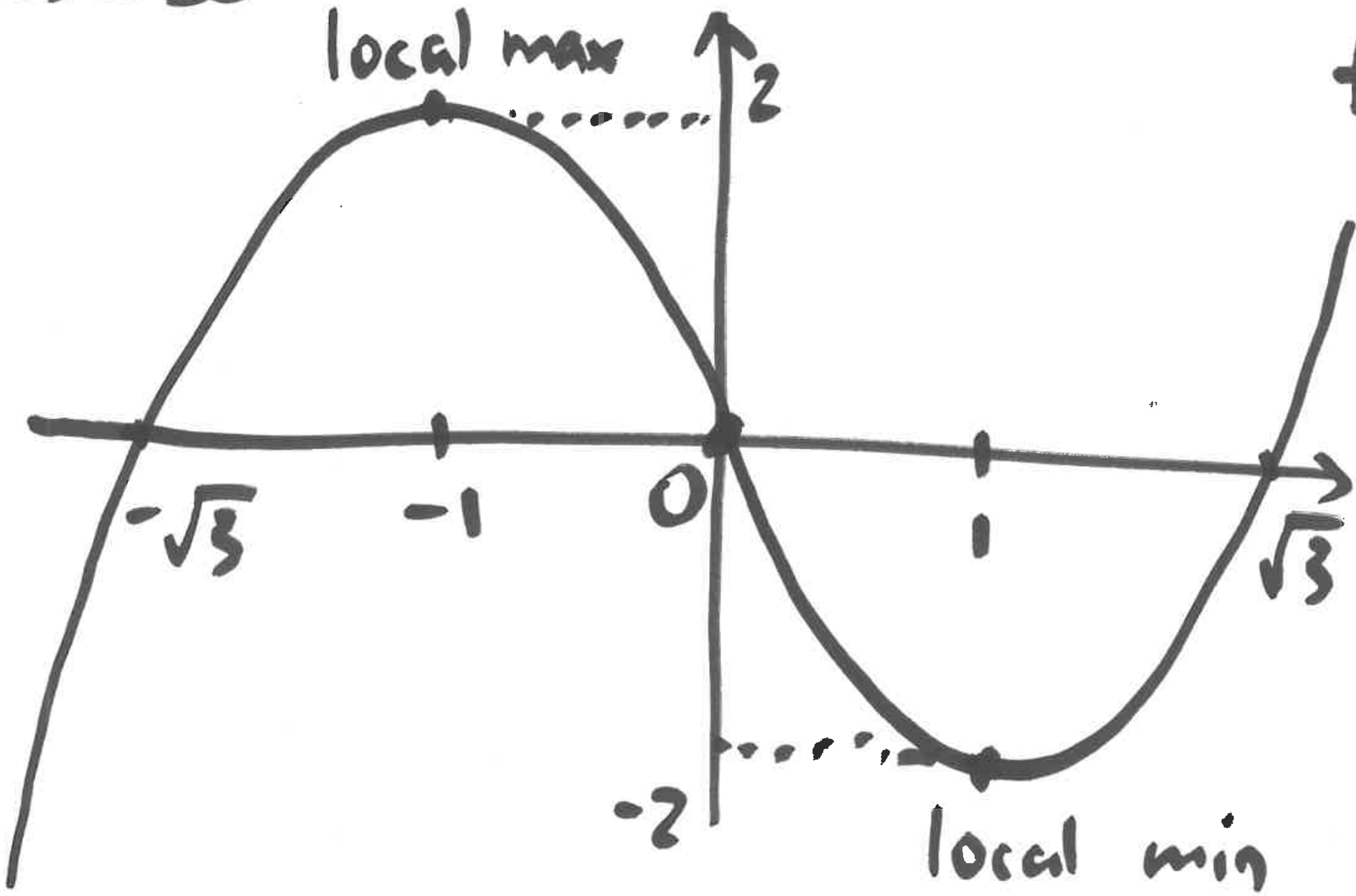
local min

$$\lim_{x \to \infty} (x^3 - 3x) = \infty$$

$$\lim_{x \to -\infty} (x^3 - 3x) = -\infty$$

$f(-1) = 2$

$f(1) = -2$



inc. interval : $(-\infty, -1), (1, \infty)$

dec. interal : $(-1, 1)$

local max : $x = -1$

local min : $x = 1$

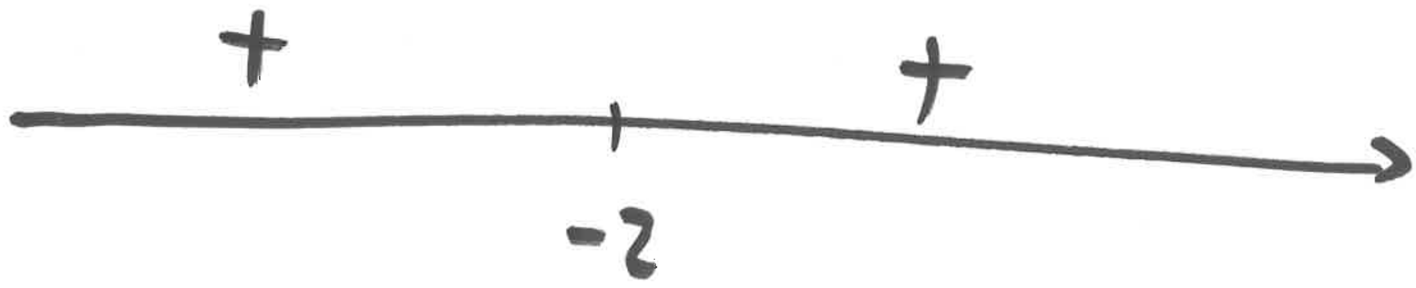
② $f(x) = \dfrac{2x-1}{x+2}$

Domain : $(-\infty, -2) \cup (-2, \infty)$

$$f'(x) = \frac{2\cdot(x+2) - (2x-1)\cdot 1}{(x+2)^2}$$

$$= \frac{2x+4-2x+1}{(x+2)^2} = \frac{5}{(x+2)^2}$$

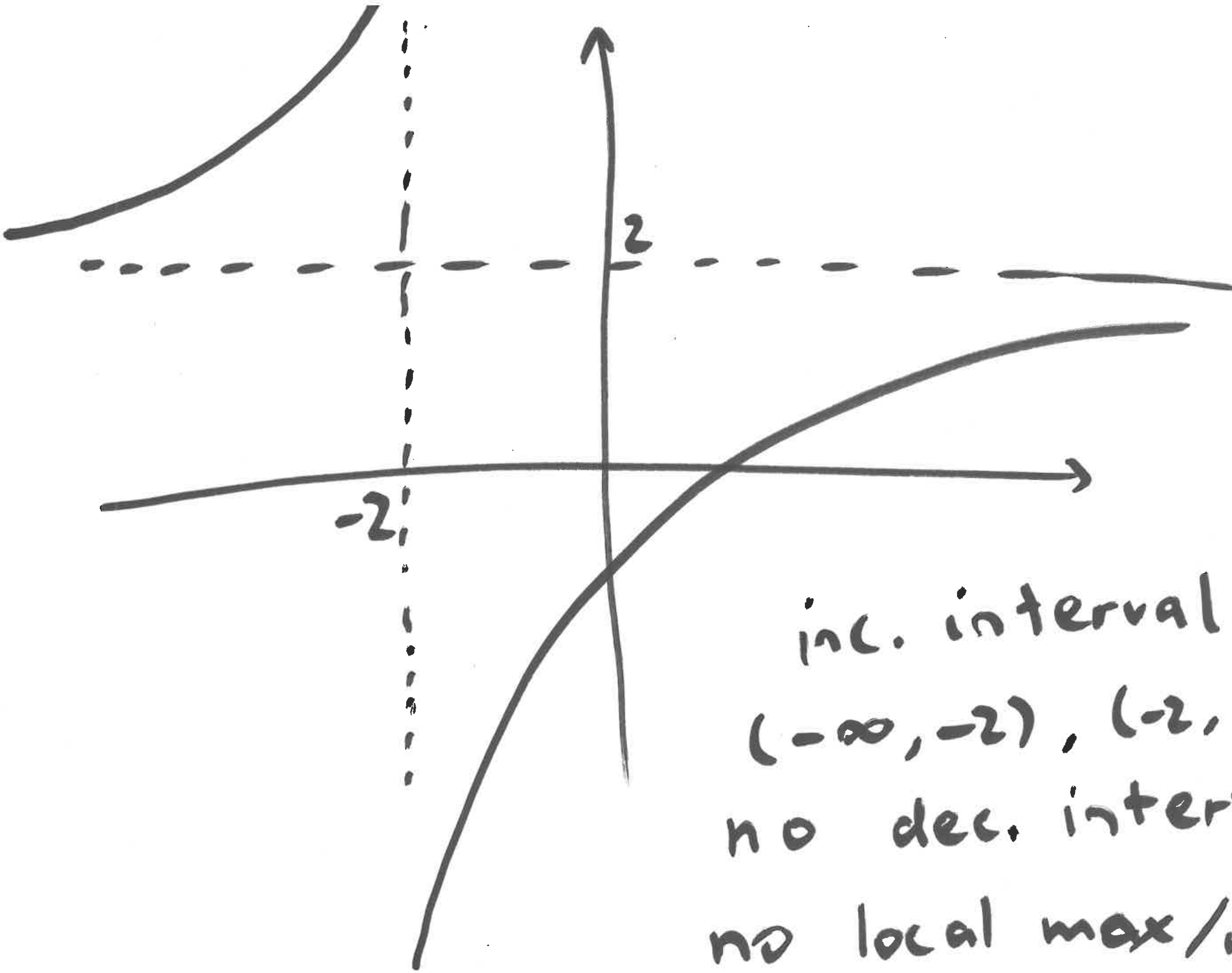
$f'(x) = 0 \Rightarrow$ no solutions!



$$\lim_{x \to \infty} \frac{2x-1}{x+2} = 2 = \lim_{x \to -\infty} \frac{2x-1}{x+2}$$

$$\lim_{x \to -2^-} \frac{2x-1}{\underset{\substack{\uparrow \\ -2.1}}{x}+2} = \infty$$

$$\lim_{x \to -2^+} \frac{2x-1}{x+2} = -\infty$$



inc. interval:

$(-\infty, -2), (-2, \infty)$

no dec. interval

no local max/min

③ $f(x) = x^4 + x^3$

$f'(x) = 4x^3 + 3x^2$

$f'(x) = 0 \Rightarrow x^2(4x+3) = 0$

$x = 0 \;,\; x = -\frac{3}{4}$

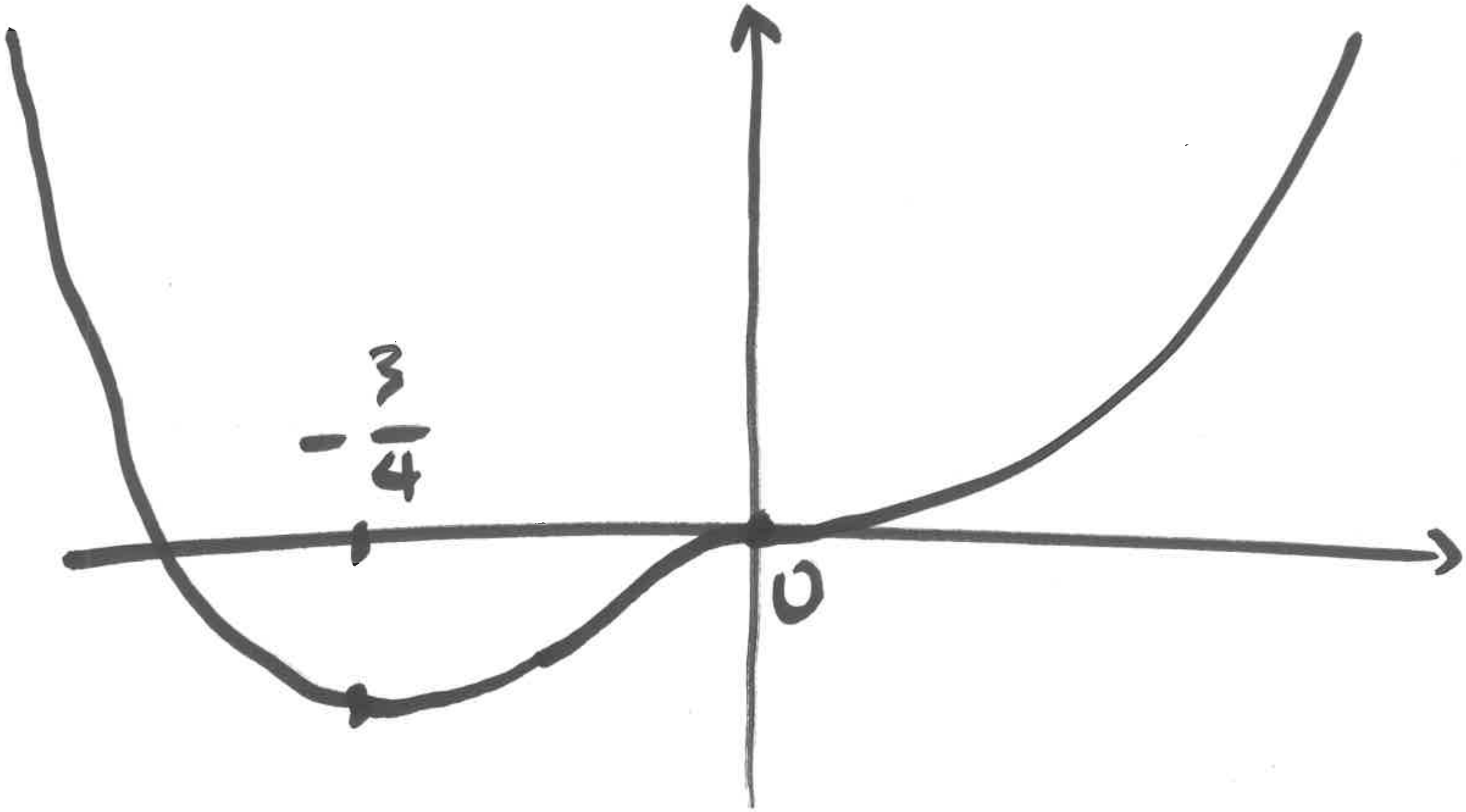
$-$ local min $+$ neither $+$

$-\frac{3}{4}$ 0

$f'(-1) = -1$ $f'(-\frac{1}{2}) = \frac{1}{4}$ $f'(1) = 7$

$4 \cdot (-\frac{1}{2})^3 + 3 \cdot (-\frac{1}{2})^2$

$$\lim_{x \to \infty}(x^4 + x^3) = \lim_{x \to -\infty}(x^4 + x^3) = \infty$$



$$f\left(-\frac{3}{4}\right) = \left(-\frac{3}{4}\right)^4 + \left(-\frac{3}{4}\right)^3 = -\frac{27}{256}$$

$$f(0) = 0$$

dec. int. : $(-\infty, -\frac{3}{4})$

inc. int. : $(-\frac{3}{4}, \infty)$

local min : $x = -\frac{3}{4}$

- For function $f(x)$ on $[a, b]$, global max/min are achieved at endpoints or critical points.

Ex 2 Find global max/min of

$$f(x) = \frac{x}{x^2+2} \quad \text{on } [-1, 2].$$

$$f'(x) = \frac{1 \cdot (x^2+2) - x \cdot 2x}{(x^2+2)^2}$$

$$= \frac{-x^2+2}{(x^2+2)^2}$$

$$f'(x) = 0 \Rightarrow -x^2+2=0$$

$$x^2 = 2$$

$$\underline{x=\sqrt{2}}, \quad \underline{x=-\sqrt{2}}$$

in [-1, 2] $\qquad$ not in [-1, 2]

ignore

endpoints:

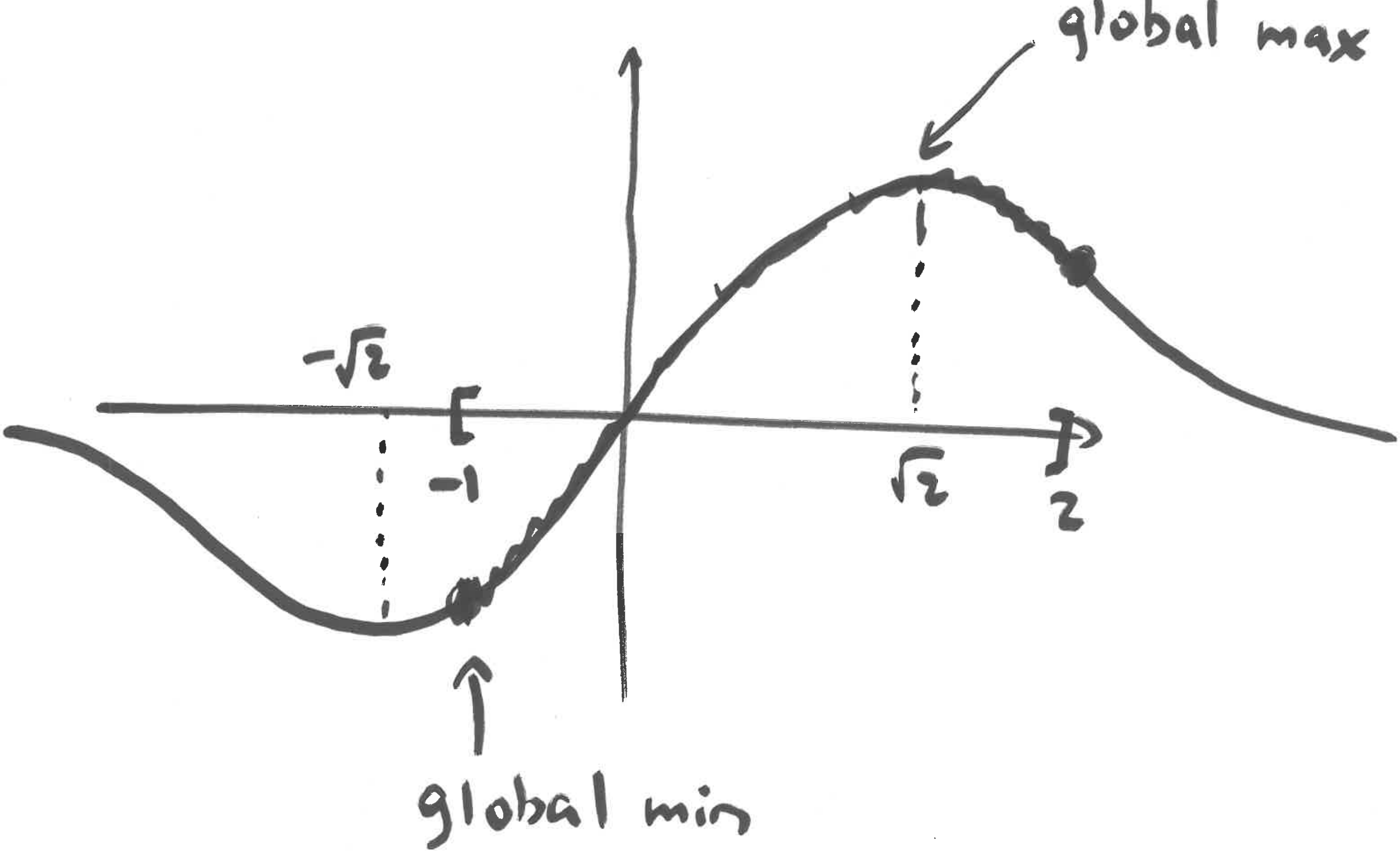
$$f(-1) = \frac{-1}{(-1)^2+2} = -\frac{1}{3}$$ global min

$$f(2) = \frac{2}{2^2+2} = \frac{1}{3} \approx 0.33$$

critical points:

$$f(\sqrt{2}) = \frac{\sqrt{2}}{(\sqrt{2})^2+2} = \frac{\sqrt{2}}{4} \approx 0.35$$

global max


global max
$-\sqrt{2}$
-1
$\sqrt{2}$
2
global min