Review

Ex 1 Compute limits

① $$\lim_{x \to 2} \frac{x^2 - 2x}{x - 2} = \lim_{x \to 2} \frac{x(x-2)}{x-2}$$

$$= \lim_{x \to 2} x = 2$$

② $$\lim_{x \to -1} \frac{x^2 + 3x + 2}{x^2 - 1} = \lim_{x \to -1} \frac{(x+1)(x+2)}{(x+1)(x-1)}$$

$$= \lim_{x \to -1} \frac{x+2}{x-1} = \frac{-1+2}{-1-1} = -\frac{1}{2}$$

③ $$\lim_{x \to \infty} \frac{x^3+2x-1}{-3x^2+2} = \lim_{x \to \infty} \frac{x+\frac{2}{x}-\frac{1}{x^2}}{-3+\frac{2}{x^2}}$$

"x^2"

$$= -\infty$$

④ $$\lim_{x \to \infty} \frac{\sqrt{x^4+4}}{2x^2-1} = \lim_{x \to \infty} \frac{\sqrt{1+\frac{4}{x^4}}}{2-\frac{1}{x^2}}$$

$$(x^a)^b = x^{ab}$$

$$= \frac{1}{2}$$

$$\sqrt{x^4+4x^2+4} = x^2+2$$

⑤ $$\lim_{x\to\infty} \frac{\sqrt{2x^3+3x}}{x^2-1} \quad \text{" } x^{3/2} \text{ "}$$

$$= \lim_{x\to\infty} \frac{\sqrt{2+\frac{3}{x^2}}}{x^{1/2}-\frac{1}{x^{3/2}}} = 0$$

Ex 2 Compute derivatives

① $(\underline{\sin x}\,\underline{\cos x})'$

$= \cos x \cdot \cos x + \sin x \cdot (-\sin x)$

$= \cos^2 x - \sin^2 x.$

② $(\ln(\boxed{1+e^x}))'$

$= \dfrac{1}{1+e^x} \cdot e^x.$

③ $\left(\left(\boxed{3x^2-2x+1}\right)^{10} \right)'$

$$= 10 \cdot (3x^2-2x+1)^9 \cdot (6x-2)$$

④ $\left(\dfrac{x+2}{3x^2-1} \right)' = \dfrac{1 \cdot (3x^2-1) - (x+2) \cdot 6x}{(3x^2-1)^2}$

$$= \frac{3x^2-1-6x^2-12x}{(3x^2-1)^2}$$

$$= \frac{-3x^2-12x-1}{(3x^2-1)^2}$$

⑤ $\left[\ln\left(\boxed{\ln x}\right)\right]'$

$$= \frac{1}{\ln x} \cdot \frac{1}{x}$$

⑥ $\left(e^{\boxed{-x^2}}\right)''$

$$= \left[\underbrace{e^{-x^2}} \cdot \underbrace{(-2x)}\right]'$$

$$= \left[e^{-x^2} \cdot (-2x)\right] \cdot (-2x) + e^{-x^2} \cdot (-2)$$

$$= e^{-x^2}(4x^2 - 2).$$

⑦ $\left(\dfrac{x^2+2x}{\sin(e^x)}\right)'$

$$= \frac{(x^2+2x)'\cdot\sin(e^x) - (x^2+2x)\left(\sin(e^x)\right)'}{\sin^2(e^x)}$$

$$= \frac{(2x+2)\cdot\sin(e^x) - (x^2+2x)\cos(e^x)\cdot e^x}{\sin^2(e^x)}.$$

Ex 3 Let $f(x) = \begin{cases} \dfrac{e^{2x}-1}{e^x-1} & x \neq 0 \\ c & x = 0 \end{cases}$

Find c such that f is continuous at $x = 0$.

$$c = \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{e^{2x}-1}{e^x-1}$$

$$e^{2x} = (e^x)^2$$

$$= \lim_{x\to 0} \frac{(e^x+1)\cancel{(e^x-1)}}{\cancel{e^x-1}}$$

$$= e^0 + 1 = 2$$

Ex 6 Find all the points of discontinuity of $f(x) = \dfrac{x^3 - 2x^2}{x^2 - 4}$

Can you define f at these pts to make it continuous?

discontinuity: $x^2 - 4 = 0$

$x = 2$, $x = -2$.

$$\lim_{x\to 2} \frac{x^3-2x^2}{x^2-4} = \lim_{x\to 2} \frac{x^2\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$$

$$= \frac{2^2}{2+2} = 1$$

we can define $f(2)=1$ to make it cont. at $x=2$.

$$\lim_{x\to -2} \frac{x^3-2x^2}{x^2-4} = \pm\infty \quad \text{DNE}$$

we cannot make f cont. at $x=-2$.

Ex 5 $f(x) = x^2 e^{-x}$. Find the tangent line to its graph at $x=-1$.

$$f'(x) = 2x\,e^{-x} + x^2 e^{-x}\cdot(-1)$$

$$= e^{-x}(2x - x^2)$$

$$f'(-1) = e^{1}(-2-(-1)^2) = -3e$$

slope

The tan. line goes through $(-1, f(-1))$

$$f(-1) = (-1)^2 e^1 = e$$

$$y - e = -3e(x+1).$$

Ex 6 The popu. of some bacteria $P(t)$ follows exp. growth model. Initially the popu. is 1000, while 2 days later, the popu. is 10000.

① Find $P(t)$

② What is the popu. at time $t=3$?

$$P(t) = 1000\, e^{kt}$$

$$P(2) = 1000\, e^{2k} = 10000$$

$$e^{2k} = 10$$

$$2k = \ln 10$$

$$k = \frac{1}{2} \ln 10$$

$$P(t) = 1000\, e^{\frac{1}{2}\ln 10 \cdot t}$$

$$P(3) = 1000\, e^{\frac{3}{2}\ln 10} = 10000\sqrt{10}$$

Ex 7 The implicit function $y(x)$ is given by

$$e^{\boxed{\sin 2x}} + \boxed{y}^3 = x^2 y + 2.$$

Find $y'(x)$. What is $y'(0)$?

$$e^{\sin 2x} \cdot \cos 2x \cdot 2 + 3y^2 \cdot y'$$

$$= 2x \cdot y + x^2 \cdot y'$$

$$(3y^2 - x^2)\,y' = 2xy - 2e^{\sin 2x}\cos 2x$$

$$y' = \frac{2xy - 2e^{\sin 2x}\cos 2x}{3y^2 - x^2}$$

$y'(0) = ?$ need $y(0)$

$$e^{\sin(2\cdot 0)} + y(0)^3 = 0^2 \cdot y(0) + 2$$

$$1 + y(0)^3 = 2$$

$y(0)^3 = 1$ $y(0) = 1$

$$y'(0) = \frac{2\cdot 0\cdot 1 - 2e^{\sin(2\cdot 0)}\cos(2\cdot 0)}{3\cdot 1^2 - 0^2}$$

$$= \frac{-2\cdot 1\cdot 1}{3} = -\frac{2}{3}$$