15.2 (continued)

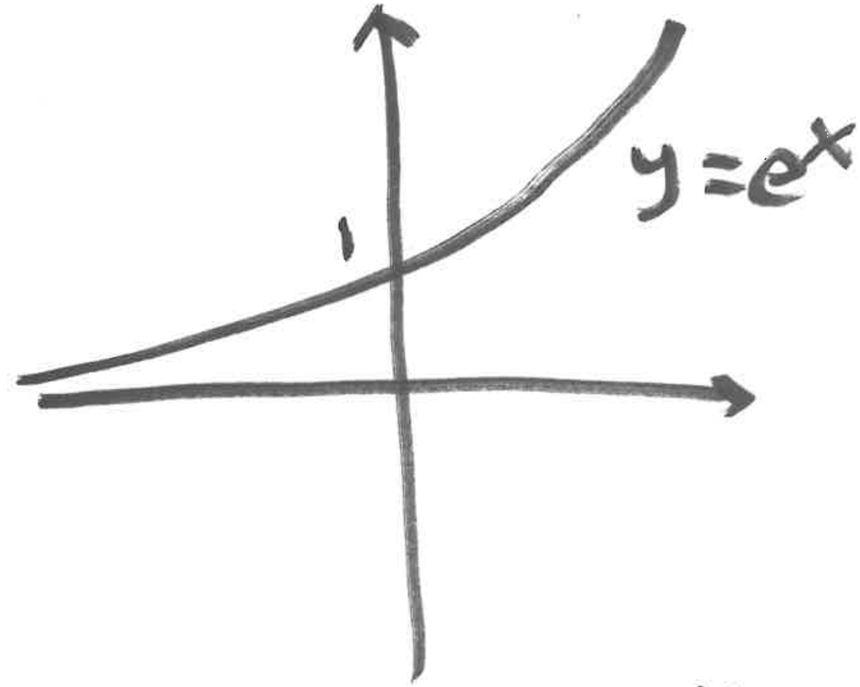
Ex 1

$$\lim_{x \to \infty} \frac{\sqrt{2x^2+1}}{x-2} = \lim_{x \to \infty} \frac{\sqrt{\frac{2x^2+1}{x^2}}}{\frac{x-2}{x}}$$

$$= \lim_{x \to \infty} \frac{\sqrt{2+\frac{1}{x^2}}}{1-\frac{2}{x}} = \sqrt{2}.$$

$$\lim_{x \to \infty} \frac{2x+3}{\sqrt{x^4-4}} = \lim_{x \to \infty} \frac{2+\frac{3}{x}}{\sqrt{x^2-\frac{4}{x^2}}} = 0$$

- $$\lim_{x\to\infty} e^x = \infty$$



$$\lim_{x\to-\infty} e^x = 0$$

<u>Ex 2</u>

$$\lim_{x\to\infty} \frac{e^x+1}{e^x-1} = \lim_{x\to\infty} \frac{1+e^{-x}}{1-e^{-x}} = 1$$

$$\lim_{x\to-\infty} \frac{e^x+1}{e^x-1} = -1$$

$$\lim_{x \to \infty} \frac{e^{2x}}{e^x + 2} = \lim_{x \to \infty} \frac{e^x}{1 + 2e^{-x}}$$

$$= \infty$$

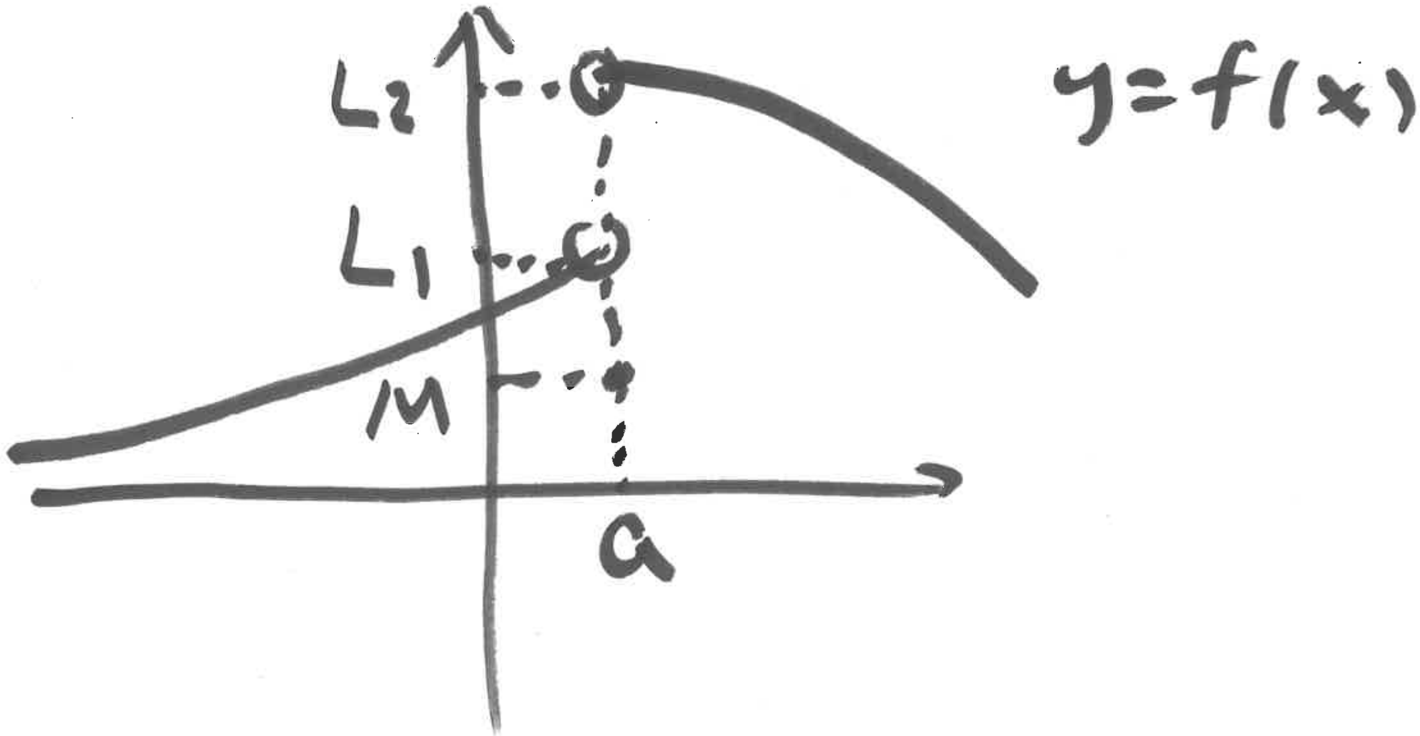
$$\frac{e^{2x}}{e^x} = e^{2x-x} = e^x$$

16.1 Right / left limits

Def $\displaystyle\lim_{\substack{x \to a- \\ (a+)}} f(x) = L$

"the left-hand (right) limit"

if $f(x)$ is arbitrarily close to L by taking x to be sufficiently close to a with $x < a$ ($x > a$)



$$\lim_{x\to a^-} f(x) = L_1$$

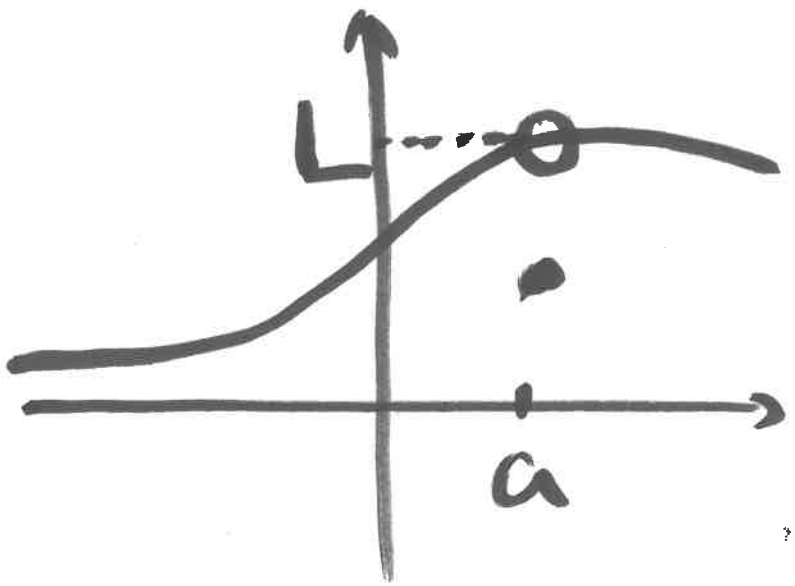
$$\lim_{x\to a} f(x) \text{ DNE}$$

$$\lim_{x\to a^+} f(x) = L_2$$

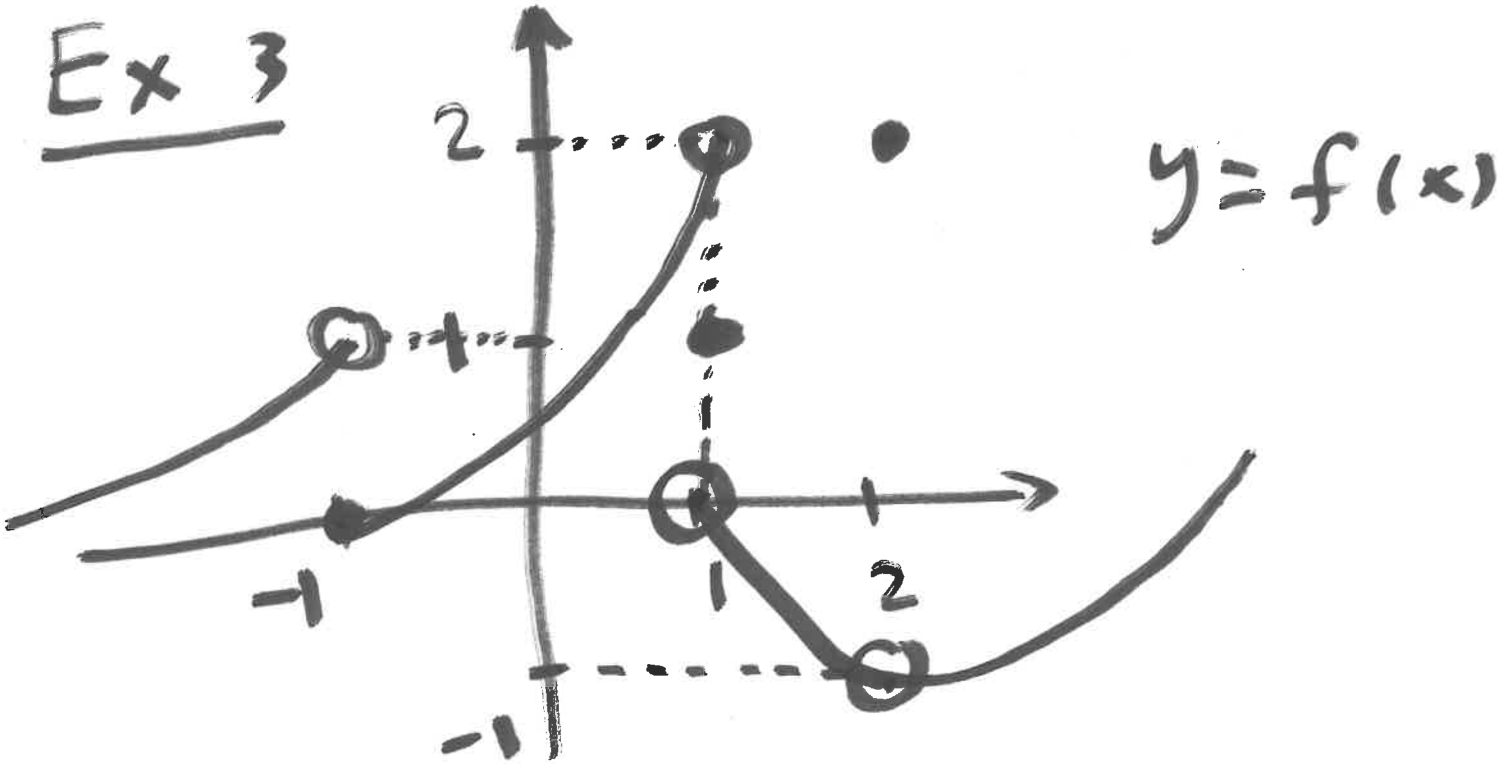
Property:

$$\lim_{x \to a} f(x) = L \iff \lim_{x \to a^-} f(x) = L$$

$$\text{and } \lim_{x \to a^+} f(x) = L$$



Ex 3



Find left/right limits as x approaches $-1, 1, 2$. At which pts does $\lim_{x \to a} f(x)$ exist?

$$\left.\begin{array}{l} \lim\limits_{x \to -1^-} f(x) = 1 \\[2ex] \lim\limits_{x \to -1^+} f(x) = 0 \end{array}\right) \neq$$

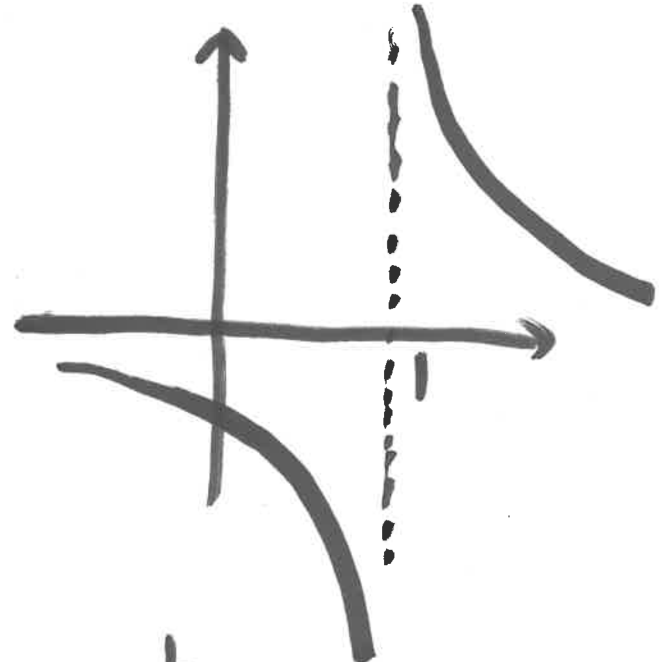
$$\left.\begin{array}{l} \lim\limits_{x \to 1^-} f(x) = 2 \\[2ex] \lim\limits_{x \to 1^+} f(x) = 0 \end{array}\right) \neq$$

$$\left.\begin{array}{l}\lim_{x \to 2^-} f(x) = -1 \\ \\ \lim_{x \to 2^+} f(x) = -1\end{array}\right) =$$

$$\lim_{x \to 2} f(x) = -1$$

Ex 4 Compute

$$\lim_{x \to 1+} \frac{1}{x-1} = \infty$$



$x-1$: small positive number

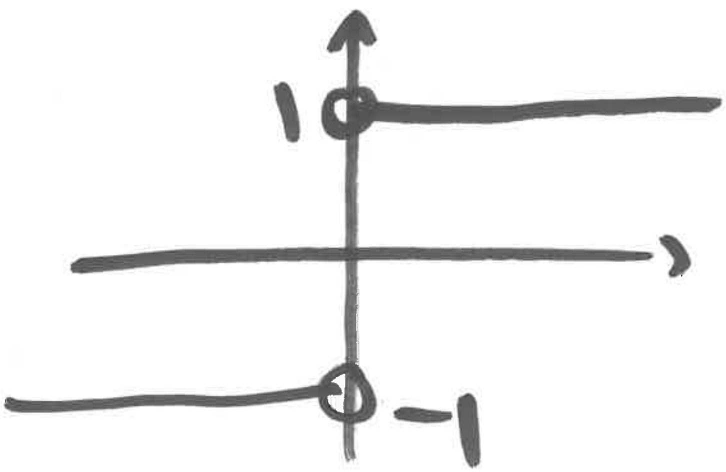
$$\lim_{x \to 1-} \frac{1}{x-1} = -\infty$$

$x-1$: small negative number

$$\lim_{x \to 0+} \frac{|x|}{x} = 1 \qquad \lim_{x \to 0-} \frac{|x|}{x} = -1$$

If, $x > 0$ $\quad \dfrac{|x|}{x} = \dfrac{x}{x} = 1$

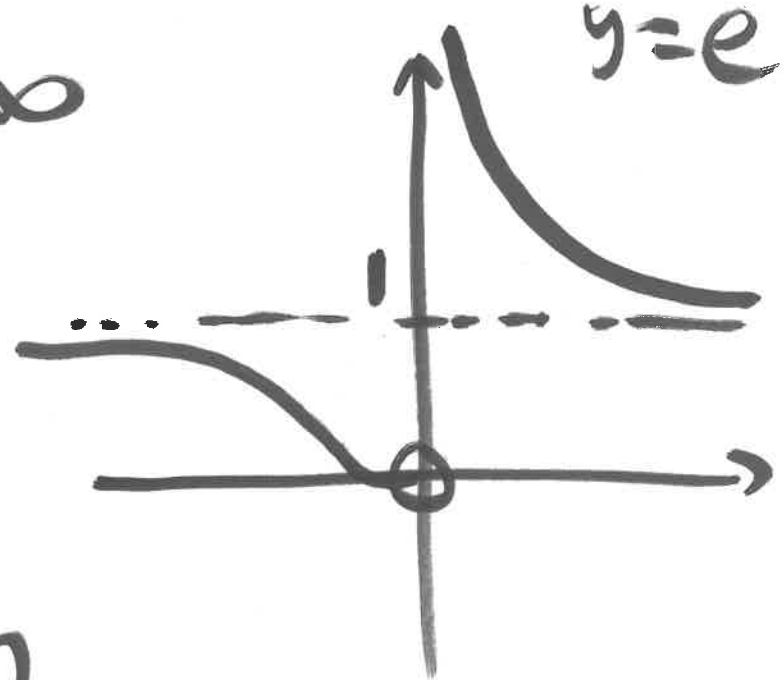
If $x < 0$ $\quad \dfrac{-x}{x} = -1$



$$\lim_{x \to 0} \frac{|x|}{x} \quad \text{DNE}$$

$$\lim_{x \to 0+} e^{\frac{1}{x}} = \infty$$

$$\frac{1}{x} \longrightarrow \infty$$



$$\lim_{x \to 0-} e^{\frac{1}{x}} = 0$$

$$\frac{1}{x} \longrightarrow -\infty$$

Ex 5 At time $t=0$ there are no cats on an island. At time $t=3$ (year), 500 cats are transported to the island. After that, the cat population grows at a rate of 20 cats per year.

(1) Write down the population of cats as a function of time.

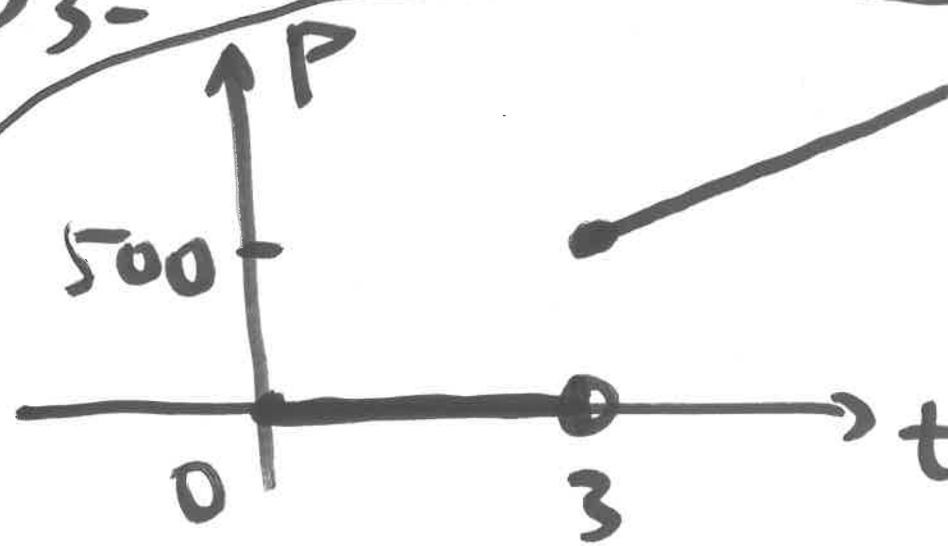
$P(t) = ?$

$$\begin{cases} \text{for } 0 \le t < 3, & P(t) = 0 \\ \text{for } t \ge 3, & P(t) = 500 + 20(t-3). \end{cases}$$

$$P(3) = 500$$

(2) Compute $\lim\limits_{t \to 3-} P(t)$,

$\lim\limits_{t \to 3+} P(t)$.



$$\lim_{t \to 3^-} P(t) = 0$$

$$\lim_{t \to 3^+} P(t) = 500$$

(3) What is $\lim\limits_{t \to \infty} P(t)$?

$$\lim_{t \to \infty} P(t) = \lim_{t \to \infty} \left(500 + 20(t-3) \right)$$

$$500 + 20t - 60$$

$$= \infty$$

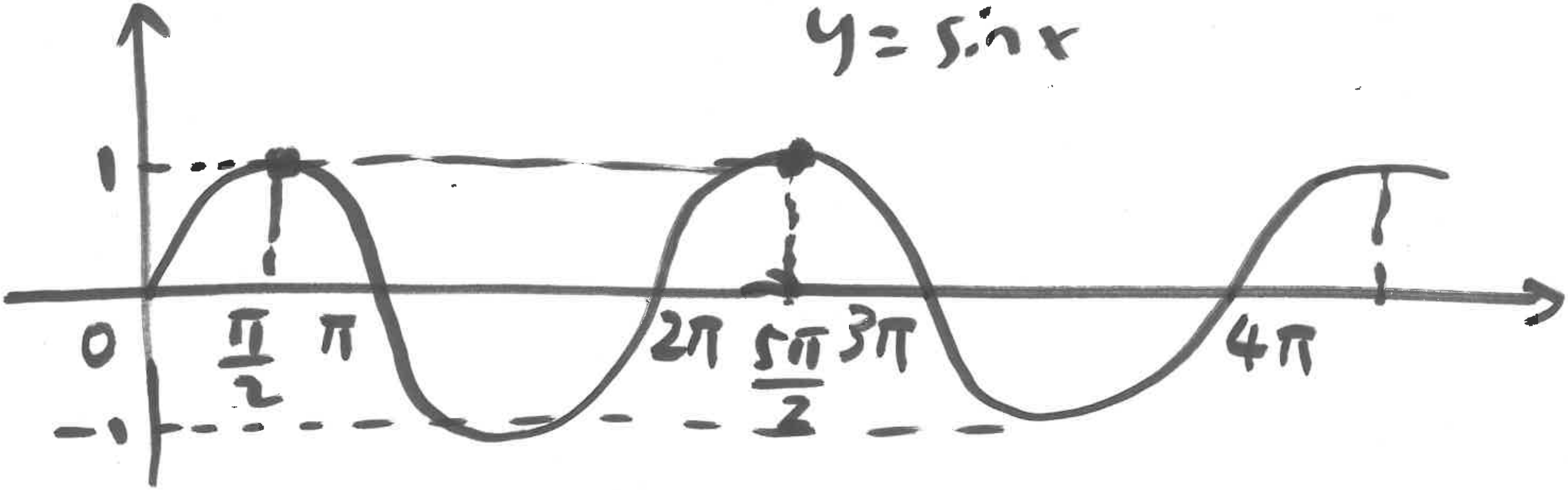
(not realistic!)

$$20t + 440$$

Ex 6 Sometimes even one-sided limit may not exist.

$f(x) = \sin \frac{1}{x}$ near $x = 0$

$$\lim_{x \to 0+} f(x)$$



$\sin(k\pi) = 0$, k integer.

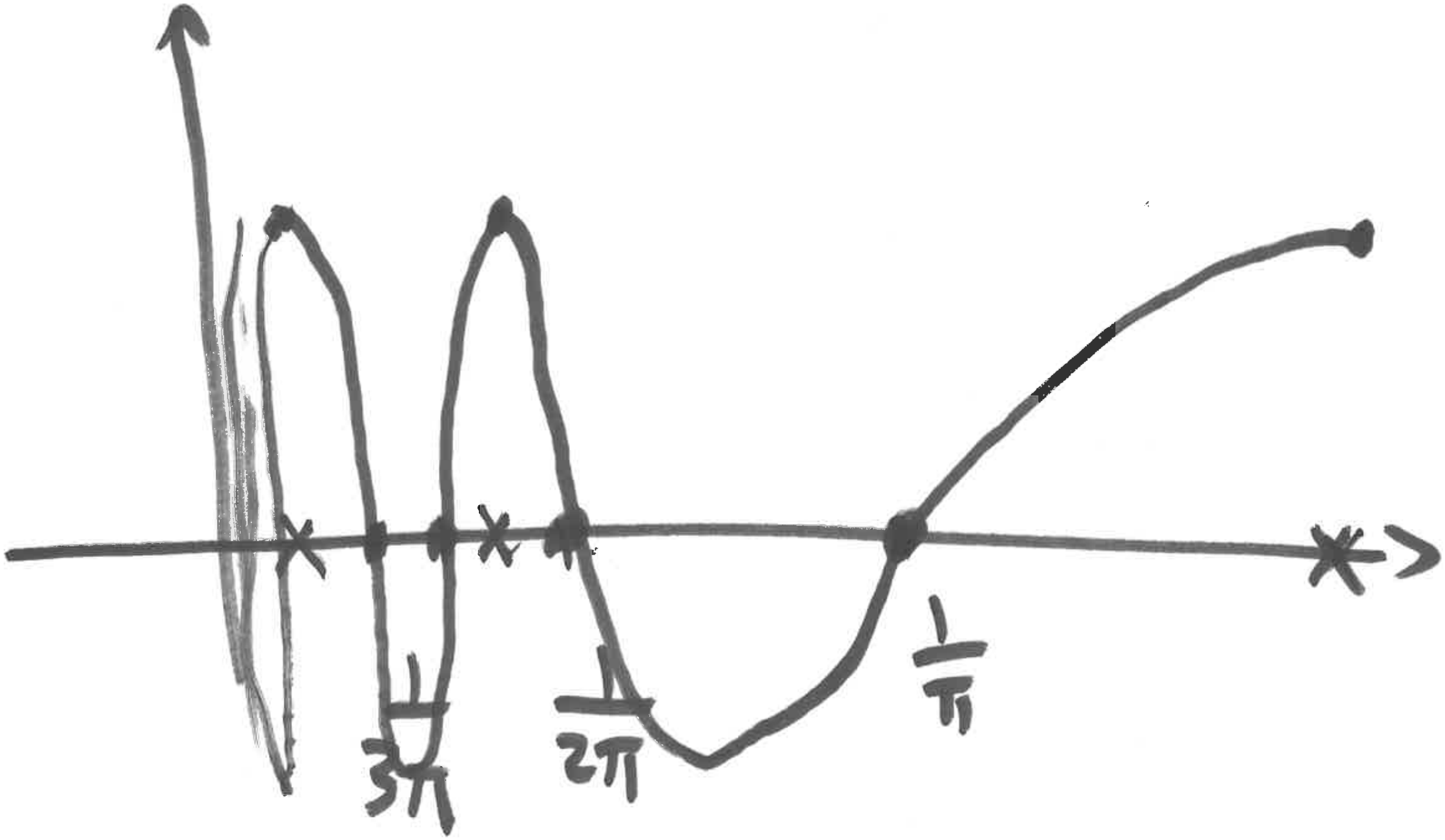
$\sin\left(\frac{\pi}{2} + 2k\pi\right) = 1$

$f\left(\frac{1}{\pi}\right) = 0$ $\qquad$ $f\left(\frac{2}{\pi}\right) = 1$

$f\left(\frac{1}{2\pi}\right) = 0$ $\qquad$ $f\left(\frac{2}{5\pi}\right) = 1$

$f\left(\frac{1}{3\pi}\right) = 0$ $\qquad$ $f\left(\frac{2}{9\pi}\right) = 1$

$\vdots$ $\qquad$ $\vdots$



$$\lim_{x \to 0+} \sin\frac{1}{x} \quad \text{DNE}.$$